The Content of the Form: Narrative Elements of Problem Solution in Physics

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Abstract

In this essay I explore the use of *theme*, *plot* and *motif* to help students construct meaning for themselves and then transmit that meaning in their written solutions. Narrative principles are also used as vehicles to introduce students to the concept of "expertise" in problem solving as well as various results from *Physics Education Research* and, most importantly, the conceptual economies of what is known in cognitive science as *the narrative effect*.

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1. Introduction

Boy meets girl, so what?

screenplay by Bertolt Brecht

The epigrammatic title of Brecht's screenplay neatly captures the cognitive economies that are the heart of this essay: three words in one actional phrase ("boy meets girl") summarize much of the canon of Western Literature (and most of the non-canonical literature as well). There have been several influential studies detailing the limited number of plots in literature: Vladimir Propp's analysis of Russian folk tales uncovers 31 basic plot components that almost always occur in all of the tales, and in the same order. (Propp, 1968) Equally influential is Tobias' *20 Master Plots* (Tobias, 2011), listed below.

Quest	Adventure	Pursuit	Rescue
Escape	Revenge	The Riddle	Rivalry
Underdog	Temptation	Metamorphosis	Transformation
Maturation	Love	Forbidden Love	Sacrifice
Discovery	Wretched Excess	Ascension	Descension

Plot refers to the structure of the story—the organization of events that make up a story and how they are related to and interact with one another. Shakespeare's *Hamlet* is a revenge plot; the *theme* of this play, however, is indecision. *Theme* is the central topic or concept of a story, and while debates rage as to whether themes can be specified by a single noun (*power*) or a verbal phrase (*power corrupts*), their number is limited, and this should come as no surprise: human experience is nothing if not repetitive, and at any rate fundamental cognitive processes demand that we categorize to reduce multiplicity:

> It is easier to organize knowledge and behavior if the vast realms of experience are subdivided; indeed, the world would quickly become unmanageable if I had to sort through every possible concept and potential course of action at every given moment. (Herman 2003, 136)

Reducing "the vast realms" by categorization is also central to the scientific enterprise: there are a limited number of archetypal problems and corresponding solutions—what Thomas Kuhn terms *exemplars* (Kuhn 1970, 187)—and successful scientists and their students have somehow come to recognize this economy. There are literally thousands of end-of-chapter problems in a standard undergraduate physics textbook, the taxonomy and solution methods of which provide the most important formative experience in science education. The successful student's ability to reduce the thousands to several exemplars is a form of tacit knowledge acquired by doing science. My objective is to use narratives to reveal this tacit knowledge to students who may otherwise continue to labour under the illusion that there really are thousands of problems—in fact there are but a few themes and a handful of plot variations.

According to Kuhn's well know historiography, scientists spend the majority, if not all, of their professional lives attempting to find answers to various problems, an activity that he characterizes as *puzzle solving.* (Ibid. 35) The puzzles themselves (and indeed their method of solution) are provided by the reigning paradigm. Science students gain implicit knowledge about scientific research as they hone their puzzle-solving skills, testing their ingenuity against problems found in standardized physics, mathematics and chemistry textbooks. The activities of problem solving and the writing of solutions are exemplary of how research proceeds and how research papers are constructed—they are central to the scientific enterprise itself.

While much work has been done in recent years on the teaching of conceptual physics[1] and problem solving there has been scant attention paid to the manner in which students communicate their conceptual understanding in written solutions. Too often students rush headlong toward the perceived end, plugging numbers into what may or may not be appropriate equations, a habit reinforced by marking schemes that privilege final answers at the expense of the communication of ideas. Those of us who continued to graduate work in science developed a feel for the conventions of good solutions, absorbing the lessons from examples in lectures and textbooks (as Saul Bellow put it, "a writer is a reader moved to emulation"[2]). We know how to construct good

[1] See, for example, McDermott et al. 1996; Eric Mazur 1997; Crouch and Mazur 2001.
[2] attributed

problem solutions, using a logical format that transmits meaningful information to the reader. As teachers we model good solution writing and ask that the students emulate these models in their own solutions. But the models are presented in a final, polished form, like Athena leaping fully-grown from Zeus' head: students are not taught strategies for developing and crafting their own solutions, and so the problem of poor communication persists, along with a fixation on the final answer. I believe that the problem-solving and solution-writing habits of successful science students and scientists can indeed be taught and that such instruction should be an explicit part of the science curriculum. If essay writing can be taught in the Humanities (and it is, often quite well) then we can and should teach the corresponding fundamental skill in the Sciences.

Two Examples

Two exemplary marking experiences frame this investigation. The first—submitted on a test—is a solution to a standard kinematics question: find the time of flight for an object launched directly upward with an initial speed v_0. The rather terse student solution looked something like the following:

$$\cancel{V_0} = V_0 - gt$$

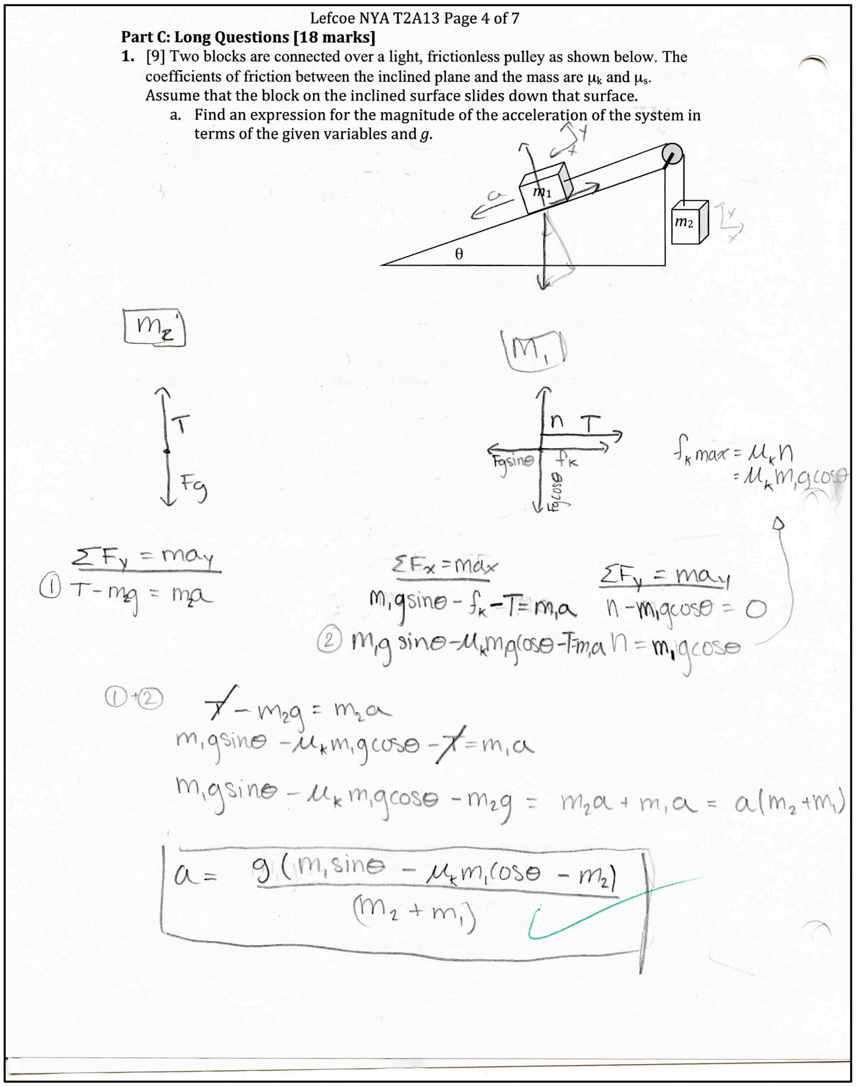
$$t = \frac{V_0}{g}$$

$$t = \frac{2V_0}{g}$$

The answer, of course, is correct: the total time of flight is indeed twice the time to the maximum height, and at this maximum height the velocity is zero. But this explanation was entirely absent, and in its absence, I discovered, I had assumed that the student well understood the concepts and method of solution involved. The student came to see me in my office to discuss other issues with his test and I pointed out the shortcomings in his solution, to which he replied something along the lines: ”Oh yah, but it’s obvious, v is zero when it hits the ground . . .” I cannot recall the rest of what was said in this brief discussion (what is termed in the literature a “think aloud protocol”), but I had heard enough to be convinced that the student deserved a mark of 0 on this problem: clearly his conceptual understanding was nil, he was merely following what are termed "novice habits" of working backward from the given data, plugging known values into an equation to obtain an answer (the distinction between expert and novice problem solvers will be discussed in detail in Section 3).

In the course material the student had no doubt encountered a solution that looked very much like his own, but such a solution would have been accompanied (one hopes) by explanations: a diagram, brief remarks, numbered equations, etc.—in short, a solution that transmits meaning rather than just a result. Within the student's brief (and now incorrect) solution there *are* fragments of conceptual clarity: he did multiply the time by 2, but what time did he think he was multiplying? The arrow showing that $v = 0$ m/s is a useful semiotic device (that is, it communicates meaning—Section 4) but unfortunately I assumed a meaning that he had neither intended nor understood.

A second marking experience is vastly preferable to the first:



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Part C: Long Questions [18 marks]

1. [9] Two blocks are connected over a light, frictionless pulley as shown below. The coefficients of friction between the inclined plane and the mass are μ_k and μ_s. Assume that the block on the inclined surface slides down that surface.

a. Find an expression for the magnitude of the acceleration of the system in terms of the given variables and g.

$\boxed{m_2}$ $\boxed{m_1}$

T, Fg n, T, Fgsinθ, f_k, Fgcosθ

$f_k max = \mu_k n$
$= \mu_k m_1 g\cos\theta$

$\Sigma F_y = ma_y$
① $T - m_2 g = m_2 a$

$\Sigma F_x = ma_x$
$m_1 g\sin\theta - f_k - T = m_1 a$
② $m_1 g\sin\theta - \mu_k m_1 g\cos\theta - T = m_1 a$

$\Sigma F_y = ma_y$
$n - m_1 g\cos\theta = 0$
$n = m_1 g\cos\theta$

①+②
$\cancel{T} - m_2 g = m_2 a$
$m_1 g\sin\theta - \mu_k m_1 g\cos\theta - \cancel{T} = m_1 a$
$m_1 g\sin\theta - \mu_k m_1 g\cos\theta - m_2 g = m_2 a + m_1 a = a(m_2 + m_1)$

$$\boxed{a = \frac{g(m_1\sin\theta - \mu_k m_1\cos\theta - m_2)}{(m_2 + m_1)}}$$

If you are a physics teacher (and had been marking 30 different student solutions to this same question for the last hour) you could probably grade this in under three seconds. Although lacking any explanatory text, the student's use of what I term *annotation*, *punctuation* and appropriate *page setup* allow one to practically inhale the solution as a whole (I will discuss *solution templates* in Section 4). Such an organized presentation of ideas is, for me, a species of narrative.

Why Narrative

Narrative is not a choice, not a chosen medium or method, rather it is intimately linked to fundamental notions of causality, how time unravels in one direction only. We seem to require narratives to make sense of our world, to understand events and our relations to them. In listening to news we need to identify fundamental plots of good versus evil, someone wronged, a crime in need of resolution, retribution, a catastrophe and its consequences.

> In everyday life people incorporate stories into a wide range of activities. Stories enable humans to carry out spontaneous conversations, make sense of news reports in a variety of media, produce and interpret literary texts, create and assess medical case histories, and provide testimony in court **(**Herman 2003, 133-4)

Although not myself a born storyteller I have noted the compelling power of narrative in the lecture hall: the ability to obtain, for some brief minutes, the absorbed attention of a class by stringing together ideas to somehow craft a story; I recall discussing the origin of number as ordinal usage in ceremony rather than cardinal usage in, for example, commerce (everyone loves a good distinction, in this case an adult education class of inner-city teenage dropouts); every semester I find an excuse to deliver my "in the beginning there was hydrogen" lecture—a scientific creation myth. I explain that the pair-wise gravitational force between hydrogen atoms in an immense primordial gas cloud gives rise to a rotational motion. Stars form by a process of accretion and, as they age, produce heavier elements, eventually leading to a *supernova* explosion that spews elements out into the void: "we are all star dust," as Isaac Asimov has said. In a few short minutes students have construed a connection between the big bang, Newton's Laws, the conservation of angular momentum, nuclear fusion and relativistic energy, and galactic, stellar and planetary formation and motions.

cognition: the narrative effect

The ability of narrative to command our attention comes as no surprise: we need stories, we love stories, they are an essential part of childhood, of our induction into the world, transmitting the skills of reading and listening and what is broadly termed culture. But more than that, cognitive psychologists have identified the fundamental role that narrative plays in our interaction, understanding and indeed construction of reality (see, for example, Bruner 1991). Narrative allows us to understand the world around us, to achieve clarity and to inflict order: narrative is a fundamental cognitive activity.

Let me say from the outset that my educational trajectory has left me somewhat deficient in the field of cognitive semiotics. But even a cursory perusal of the literature leaves one convinced of the power of narrative to organize our experience and ideas: from a plethora of possible citations to justify the cognitive priority of narratives I have chosen the following declaration by renowned cognitive psychologist Jerome Bruner:

> . . . it is very likely the case that the most natural and the earliest way in which we organize our experience and our knowledge is in terms of narrative form." (Bruner 1996, 121)

Why does narrative play such an important role in cognition? To paraphrase George Bernard Shaw's comment about economists, if you lay all narrative theorists end to end they would not reach a conclusion:

> It remains somewhat of a mystery why narrative text is so easy to comprehend and remember. Perhaps it is because the content of narrative text has such a close correspondence with everyday experiences. Perhaps it is because the language of oral conversation has a closer similarity to narrative text than other discourse genres. Perhaps it is because there are more vivid mental images, or a more elegant composition of the conceptual structures. (Graesser 2002, 16-17)

Whatever the cause, there is much evidence for narrative's cognitive power:

> . . . there is research showing that narrative passages are read faster, comprehended better, and tend to be more absorbing than expository passages and perhaps than other genres as well . . . a good narrative can increase the plausibility and persuasiveness of information presented, a finding that would be important for science education, which places considerable emphasis on information. It has also been found that narrative passages positively affect memory . . . and that readers apply themselves more when reading narrative compared to expository prose. (Norris et al. 2005, 553-4)

> . . . we organize our experience and our memory of human happenings mainly in the form of narrative—stories, excuses, myths, reasons for doing and not doing, and so on." (Bruner 1987, 4)

> . . . skill in narrative construction and narrative understanding is crucial to constructing our lives and a "place" for ourselves in the possible world we will encounter. (Bruner 1996, 40)

narratives in science

Narrative is not normally associated with the sciences, but nor is it exclusive to any one domain: many humanistic fields of inquiry, from Film Studies to Musicology and—as one would expect—Literary Studies, use narrative theory in the analyses of film, music and literature, respectively (see, for example, Branigan 1992; Mauss 1991). Narrative is ubiquitous, a necessary part of the human experience:

> Narrative has existed in every known human society. Like metaphor, it seems to be everywhere: sometimes active and obvious, at other times fragmentary, dormant, and tacit. We encounter is not just in novels and conversation but also as we look around a room, wonder about an event, or think about what to do next. One of the important ways we perceive our environment is by anticipating and telling ourselves mini-stories about that environment based on stories already told. Making narratives is a strategy for making our world of experience and desires intelligible. It is a fundamental way of organizing data. (Branigan 1992, 1)

In what sense can Science use narratives? I believe there to be three fundamental areas in which narrative processes are involved: scientific theories, how we discover and teach these theories, and the "spectrum of tasks that are called problems" (Maloney 2011, 3). For students and teachers alike the three are inevitably fused: I must briefly dispense with the first two, each of which could easily occupy a lifetime of research, for the third is the very subject of this essay.

The fundamental cognitive role of narrative leads one to expect that it play a vital role in the creation, elaboration, transmission and understanding of scientific theories: "narrative is indivisibly fused with the theoretical enterprise . . . all theories tell a story." (Hoffman 2005, 310) Bruner is interested in enlivening science education by explicitly acknowledging the role of narrative in the creation of finished scientific theories:

> The process of science is narrative. It consists of spinning hypotheses about nature, testing them, correcting the hypotheses, and getting one's head straight . . . we play with ideas, try to create anomalies, try to find neat puzzle forms that we can apply to intractable troubles so that they can be turned into soluble problems" (Bruner 1996, 127)

To this end he calls for the emphasis in the classroom to be placed "back on the process of science problem solving rather than upon the finished science and "the answers." (Ibid.) Millar and Osborne (1998, 2013) propose that science education "make greater use of one of the world's most powerful and pervasive ways of communicating ideas—the narrative form." Narrative excels at "communicating ideas, and in making ideas coherent, memorable, and meaningful."

(Ibid.) The narrative emphasis on interrelated sets of ideas stands in stark contrast to the conventional delivery of science curriculum where one focuses on ideas in isolation, thereby obscuring the major themes of the science.

And now to the third usage of narrative in science, namely problem solving: What precisely do I mean by the term *narrative* in the context of problem solving and solution writing? Does it refer to the form of a final written solution or the process of solving the problem? As will be seen it is impossible to separate these two processes in science, and indeed we need not and should not do so: "the word 'narrative' may refer to either the product of storytelling or to its process of construction." (Branigan 1992, 3) Thus in this study I will term the entirety—from reading the problem to penning a complete solution—a *problem solution*. In the following I propose the use of narrative as means to teach effective problem solution.

2. Narrative Problem Solutions

The approach outlined here provides a framework for studying how stories enhance core problem-solving abilities in a variety of communicative contexts, nonliterary as well as literary.[3]

> *David Herman, How Stories Make us Smarter: Narrative Theory And Cognitive Semiotics*

Problem Solution in the Lecture Hall: The Lie

problem versus exercise

What do we mean when we refer to a problem? What is problem solving? Sometimes a statement of the obvious is a useful starting place: "problem solving is what you do when you don't know what to do." (Wheatley 1984) And what of the term *problem*? There exist any number of definitions of this term, and while no one is universally accepted the following, from Hayes, captures many of the issues that occupy us here:

> Whenever there is a gap between where you are now and where you want to be, and you don't know how to find a way to cross that gap, you have a problem. (Hayes, 1980; as quoted in Bodner 1987, 513)

Put another way (see Mayer, 1992), the problem is presently in some state, it is desired that the problem be in another state and there is no direct, obvious way to accomplish the change. Many end-of-chapter questions put students in just such a cognitively productive predicament, but not all of them do, leading to the fruitful distinction between a *problem* and an *exercise*. Simply put, if you know what to do, it's an exercise, possibly what is derisively known as a *plug-and-chug* question. Textbook authors and publishers use this distinction to organize end-of-chapter work into *Exercises* and *Problems*, and rare is the textbook that does not follow this sequence.

When confronted with a real problem, then, initially one does not know what to do. How should one begin? I've always told students: "If you don't know what to do, do what you know

3 Hermann, 155

and see where it leads." To be sure more rigorous procedures for problem solving abound in the literature, and of these the mathematician Polya's four-step procedure is a frequent point of departure (Polya 1945):

1. Understanding the problem: here the solver gathers information.
2. Devising a plan: when this phase is reached the problem solver tries to use past experience to find a method of solution.
3. Carrying out the plan: the problem solver tries out the plan of solution.
4. Looking back: during this final phase the problem solver tries to check the result by using another method or by seeing how it all fits together.

The pedagogical issue I would like to address is as follows: what is a *real* problem for students has become, for teachers, a mere exercise:

> Status as a problem is not an innate characteristic of a question; it is, rather, a subtle interaction between the question and the individual trying to answer the question. It is a reflection of experience with that type of question more than intellectual ability. (Bodner 1987, 513).

When we model problem solving in a lecture we are rather showing an algorithm for solving an exercise and not following either Polya's procedure or our own expert procedures when confronted by what is truly a problem.

the lie (the gap between problem solving and presenting the solution)

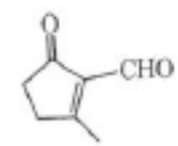
And so it is that teachers tend to present problem solutions in lectures as a *fait accompli*: in other words the teachers themselves have already performed the conceptual work required to reduce the problem to an exercise and then, in what I term *The Lie*, present this exercise to the class as a model of problem solution. The *post facto* nature of lecture problem-solving is summarized by Bodner:

> In virtually every recitation section, the students asked the TA to do this problem. Time and time again, the TA's told the students that the problem could be worked more or less like this:
>
> Start by converting grams of AgBr into moles of AgBr. Convert moles of AgBr into moles of Br, and then convert moles of Br into grams of Br. Subtract grams

> of Br from grams of indium bromide to give grams of In. Convert grams of In into moles of In. Then divide moles of Br by moles of In to get the empirical formula of the compound.
>
> During the next staff meeting, I asked my TA's to stop lying to the students. I suggested that (1) the technique for solving this problem that they had presented to their students had little to do with the process that they had used to solve the problem for the first time, (2) they had confused the process used to solve exercises with the process used to solve problems, and (3) the description given above was an algorithm for solving similar questions that they had constructed after they had solved this problem. (Ibid.)

To contrast *exercise* and *problem* Bodner poses a question that would give pause to most chemical researchers (Ibid.):

> Design a synthesis for the following compound.



For a similarly taxing problem in physics, consider the following[4]:

> An hourglass sits on a scale while sand flows from its upper to lower chambers. Does the scale register the net weight of the hourglass (housing + sand) or something else? What exactly does it measure?

Bodner continues:

> While working the question given above, you undoubtedly spent a considerable amount of time on the stage Polya described as "understanding the problem." What is less certain is whether you went through a separate stage in which you devised a plan to solve this problem before carrying out the plan. The steps that many people go through while working a "problem" such as this might be represented more or less as follows. You began by reading the problem, perhaps more than once. You then wrote down what you hoped was the key information, reread the question or a part of the question, drew a picture to help represent the question, and then tried something. Then you tried something else, and looked at where this led you. By gradually exploring or playing with the question you got closer and closer to the answer. It's possible that you never fully "understood" the question until you had an answer.

Here Bodner has captures the essence of problem solution. Yet this is not what happens in lectures:

[4] Thanks to Dr. Jérémie Vinet of Marianopolis College for suggesting this problem.

> If you compare this reconstruction of the steps many of us take while solving a problem with the description we all too often give our students of how we solved the problem, you may understand why I asked my TA's to stop lying to the students, and you may also understand the role of algorithms in solving exercises versus problems."(Ibid, 514)

Consider the description of standard problem-solving instruction:

> The most common approach involves exhibiting illustrative examples of problem solutions and then providing students with practice in solving similar problems. Occasionally some teachers suggest also a few helpful rules of thumb, while other teachers advocate predominantly student learning by independent discovery. (Reif 1981, 310)

This author continues to decry the methodology:

> Such approaches are neither too effective nor efficient in furthering students' learning of problem-solving skills, nor do such approaches lead to a cumulatively growing body of reliable knowledge about effective teaching methods. Indeed, teaching methods based predominantly on examples, practice and discovery are more primitive than those used in simpler domains (e.g. playing musical instruments or performing in sports) where many instructors use explicit teaching methods based on a systematic analysis of underlying component skills. (Ibid.)

As one actively involved in sports and music I do object to the term "simple," but that caveat aside it does seem to be the case that reducing to component skills is the basic bread and butter of teaching in these domains. Why, I wonder, do we shun this strategy in science education? In fact I feel we barely even alert students to the existence of such component skills.

The Skills We Value, The Habits that Work

the murder board

When faced with a problem (as opposed to an exercise) researchers and graduate students in Science, either alone or with their peers, invariably move to a whiteboard (or blackboard) and begin sketching, graphing, gesturing and discussing. In my experience, when discussions begin seated the interlocutors invariably rise to the board and begin this multimodal communication, this core habit, a first-response maneuver that I term the ***murder board***[5] approach. At some point

[5] Taken from crime investigation literature and films, the murder board is typically a vertical surface on which evidence and clues are displayed. The format of the items is multimodal: text,

in the discussion the particulars of the problem at hand are long forgotten, the problem has been translated through various stages of representation, finally distilled to an essential question physical theory—the crucial shift from the particular to the universal has occurred and the problem solution is nearly complete.

The above photograph of a murder board[6] demonstrates the type of messing about that occurs during problem solution: information and concepts are represented not as linear text or numbers and formulae to be processed algorithmically but rather by using multimodal resources: specialized diagrams, equations, text etc. Of course this work need not occur on a whiteboard—a scrap of paper will do.[7] Once this problem solving has been brought to a successful close one can write a coherent solution. The lie discussed above is in presenting the coherent solution without

photographs, sketches, arrows, etc. Invariably working the material on the board produces insights and eventually the solution to the problem (the crime).

[6] taken from http://cf.foreveryoungadult.com/_uploads/images/Pretty_Little_Liars_S04E03_KissThemGoodbye_net_0708.jpg

[7]. . . the Einsteins were taken to the Mt. Wilson Observatory in California. Mrs. Einstein was particularly impressed by the giant telescope. 'What on Earth do they use it for?' she asked. Her host explained that one of its chief purposes was to find out the shape of the Universe. "Oh", said Mrs. Einstein, "my husband does that on the back of an envelope. - Bennett Cerf in *Try and Stop Me.*

the intellectual work required to achieve lucidity, work done at the murder board.

other habits

I characterize the murder board activity as a habit, but it could just as well be termed a skill. Below is a partial list of similarly valued skills/habits in the sciences.

- The ability to provide coherent explanations and justifications using written language, mathematics, sketches and various other abstract representations (such as free-body diagrams, energy level diagrams, graphs, charts, tables, etc.).
- The ability to perceive and manipulate correspondences between these various modes of representation and use these correspondences as cues in problem solution.
- The recognition of problem types. That is, the ability to recognize the similarities in seemingly disparate problems, to reduce the plethora of end-of-chapter problems to a few exemplars, allowing us to more productively focus our intellectual efforts.
- The ability to transform a novel problem into one of these exemplars, hence reducing the problem to an exercise.
- The ability to get unstuck, to find one's way out of a problem after initial failure.

Some students do develop these crucial habits and skills independent of lecture and textbook presentation (in fact, most scientists fall into this category). These students have found efficient ways to organize their knowledge and integrate material presented throughout the course; to see not individual problems but classes of problems; to use physical concepts as their opening salvo in problem solution. Many other students not only have not developed these skills but also are scarcely aware of their existence:

> Common teaching practices usually pay far too little attention to issues of knowledge organization. Thus material is usually presented sequentially, chapter by chapter or lecture by lecture, so that students themselves must somehow try to integrate all this accumulating knowledge into a coherent organization facilitating flexible knowledge use. (The task of creating such an effective organization is a substantially difficult undertaking which most students are ill prepared to carry our without outside assistance.) Furthermore, arguments or problem solutions, presented in books or classrooms, are usually exhibited in the form of linear sequences of steps. Such a presentation may be impeccable from a

> purely logical point of view. However, unlike a more hierarchical organization, it is *not* well designed to help students remember or apply such knowledge. (Reif 1981, 316)

The aim of my proposal is to alert students to the proven habits and skill sets of experts, to help them organize their knowledge in a way that benefits problem solution and overall comprehension and, crucially, to give them templates to communicate meaning in their solutions, all of this using narrative: "To turn a set of events into comprehensible discourse requires the act of narration,"[8] what the literary scholar Hayden White calls the "solution to the problem of how to translate knowing into telling" (White 1981, 1) For me the transmission of meaning is paramount, a way of "telling" what we know and have learned in the course of our problem solution.

My Proposal, In Brief

> . . . even mathematical proofs, with one step following another toward an inevitable conclusion, exhibit something of the dynamics of plot and closure." (quoted in Branigan 1992, 12)

In the Introduction I discussed the cognitive power of narratives, what is termed "the narrative effect." I propose to draw on narratives in two manners, one seemingly superficial (yet useful), the other, I believe, more profound.

the genre of crime fiction

I liken the quintessentially scientific habit of messing about on a whiteboard to the unraveling of the sequence of events in a murder mystery, culminating in the climactic scene (in the classic Agatha Christie format) where the detective assembles the various participants in the drama and presents his or her solution: "*The Butler did it in the garden with the kitchen knife.*" A solution is not a mere chronicle (a list of events and dates), it is not meant to entertain (though it could), there is no need for poetic license: a solution should have the clarity, simplicity and full explanatory power of the detectives exposition—who, what, where, when and why (as required).

[8] http://miriamposner.com/blog/history-narrative-and-the-body/

The murder board activity is the initial stage of problem solving, ending with a cogent statement of the solution in the solution-writing process. These two stages—solving the mystery and presenting the findings—provide a useful model to wean so-called novice problem solvers from their tendency to approach problems as they would exercises. But here I'm aiming for a more profound realization of the narrative effect in physics problem solution.

theme, plot and motif in problem solution

Detectives present the answer (the guilty party) but also meticulously retrace their reasoning to fully justify the conclusion as *the* solution to the crime (and also to fulfill certain literary requirements of closure). Similarly in science education at the post-secondary level we are interested not merely in the *answer* but rather more so in the method of solution and reasoning: in short, the *solution*—the what, where, when and, most importantly, why. We would like well-written solutions that communicate ideas, and it is here, at a deeper, structural level that I wish to draw upon the seemingly universal communicative power of narratives. In problem solution the move from the particular (the problem at hand) to the universal (the physical principle at stake), the recognition of problem categories (to be discussed in detail below) brings to mind the distinctions between *story*, *plot* and—at a more fundamental level—*theme*. Deep analogy or not, I will argue that recognition of these narrative elements will stimulate and organize students' efforts and ideas for problem solution. To this end I have selected terminology from literary studies which carries both many readily-available associated meanings and exemplars for students while remaining somewhat faithful to literary theory (where, at any rate, there seems to be no uniform consensus concerning definitions).

Story

- This is the particular problem at hand. Consider the following:

When jumping, a flea accelerates at an astounding 1000 m/s^2, but over only the very short distance of 0.42 mm. If a flea jumps straight up, and if air resistance is neglected (a rather poor approximation in this situation), how high does the flea go? (Problem 2.47, from 2012, 66)

Theme

- Theme is the *universal* to the story's particular, the physical principle at stake. There are only a handful of such principles in each course, listed in the table below for *Introductory Mechanics*. The above problem can be solved with one theme (Kinematics With Constant Acceleration) or two (Kinematics With Constant Acceleration and Conservation Of Mechanical Energy)

Plot

- Plot refers to the organization of events or ideas, not of the literal problem (the story) but of the solution. There are a finite number of plot categories; the plot of the above example is a bipartite structure where the final state of part one becomes the initial state of part two. Students encounter this very plot structure throughout courses in mechanics, vibrations and waves, electricity and magnetism and introductory modern physics: in short, in physics.

Motif

- Midway between deep structure (theme: Work/Conservation Of Mechanical Energy) and the superficial (story: block on an inclined plane), motifs are recurring particulars of physical or mathematical significance. In the flea jump example above the motif is what I term *extrema* (in this particular case, speed and kinetic energy are zero at maximum height).

Below is a summary table of correspondences.

literary term	literary definition	examples	physics	examples
story	A particular realization of the plot. The temporally-sequenced events in a narrative.	Particular realizations of the coming of age plot: *Huckleberry Finn; Black Swan Green*.	The actual problem or the surface characteristics of the problem.	The type of objects in the problem (inclined planes, pulleys, springs), the physical terms mentioned (friction, center of mass) or the relations among objects (block on an inclined plane).
theme	The central topic or concept (eg Hamlet: indecision) or what the story says about the topic (with great power comes great responsibility). There is often more than one theme in a story.	love jealousy power corrupts	Physical concepts or principles.	Kinematics With Constant Acceleration Dynamics Inertia Newton's Laws Torque Work/Conservation Of Mechanical Energy (I call this "Conservation of Energy . . . or Not") Conservation Of Linear Momentum Conservation Of Angular Momentum Superposition/Interference (to name but one from another course)
plot	The organization of events that make up a story, how they are related, structured and interact with one another. *e.g. Hamlet*: revenge	quest triangle coming of age	The structure of the written solution.	bipartite $(\text{final state})_{\text{part 2}} = (\text{initial state})_{\text{part 1}}$ flea jump example above ballistic pendulum (see below) difference *e.g.* interference generated by path difference, phase difference upon reflection change given either the final or initial state, find the other conservation determination of state at some instant (*e.g.* statics) constraints interacting objects (*e.g.* Newton's Third Law) Problems in two dimensions yield two equations with two unknowns etc.

literary term	literary definition	examples	physics	examples
motif	A recurring object, concept or structure, something more concrete than theme.	References to blood and water in Shakespeare's play *Macbeth*. The green light in *The Great Gatsby*	Concepts (weightlessness, extrema) Constraints (rope on pulley without slipping; within the visible range) Mathematical structure, methods or techniques	equilibrium dividing two equations initial and final states symmetry extrema (what's zero) proportionality (*e.g.* $I \propto A^2$) various tricks of the trade (*e.g.* simplify problem or derive new information by running time backwards) geometry: *e.g.* location of pivot with respect to the centre of mass

And where is narrative in all of this? In literature and film, possibly also music, narrative is the collection of all of the above devices, and more besides (characters, climax, narrator, etc.), used to transmit the particular story and the universal meanings, morals and emotions. Below are examples of these narrative elements in problem solution.

a few examples

Example 1. Interference in two dimensions

- story: *Consider two speakers separated by 40.0 cm and driven by the same oscillator. A listener is located 2.00 m directly in front of one speaker. For which frequencies in the audible range (20.0 Hz—20.0 kHz) does the listener hear a maximal signal?*
- theme: superposition/interference
- the narrative (a skeletal outline of the solution)
    - theme: waves generated in phase at the speakers may no longer be in phase for the listener
    - plot/motif: interference due to path difference

For maxima in sound intensity set

$$\Delta r = m\lambda_m = m\left(\frac{v}{f_m}\right)$$

Thus $f_m = \dfrac{mv}{\Delta r}$.

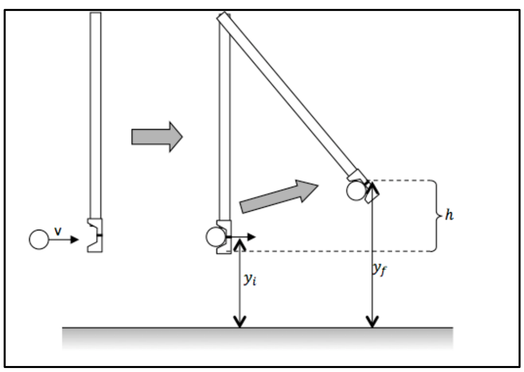
- motif: two constraints (maxima, within the audible range)

  use the second constraint to find the set of allowable frequencies

  $$20Hz \le f_m \le 20kHz$$

  This is a type of solution we value, yielding an algebraic expression in terms of *m* before determining numeric values. Thus one can easily distinguish conceptual from arithmetic errors.

Example 2. the ballistic pendulum[9]



- story: There are many variations on this story: for example, one could be given the final height *h* at which the apparatus comes to rest and be asked to determine the initial speed *v* of the ball.
- the narrative (a skeletal outline of the solution)

  This problem can only be solved by recognition of its bipartite structure: novice problem solvers are stymied by trying to find and then plug numbers into equations for the final height *h*. Recognition of the structure reduces the problem to two relatively straightforward exercises.

[9] Diagram by Dr. Celine Lebel, Marianopolis College

- theme: Conservation of Linear Momentum, Conservation of Energy
- plot/motif: bipartite structure, $\text{final}_{\text{part 1}} = \text{initial}_{\text{part 2}}$ (same as for the flea jump and untold other problems)

    Part 1: apply the principle of Conservation of Linear Momentum to the collision between the ball and the block

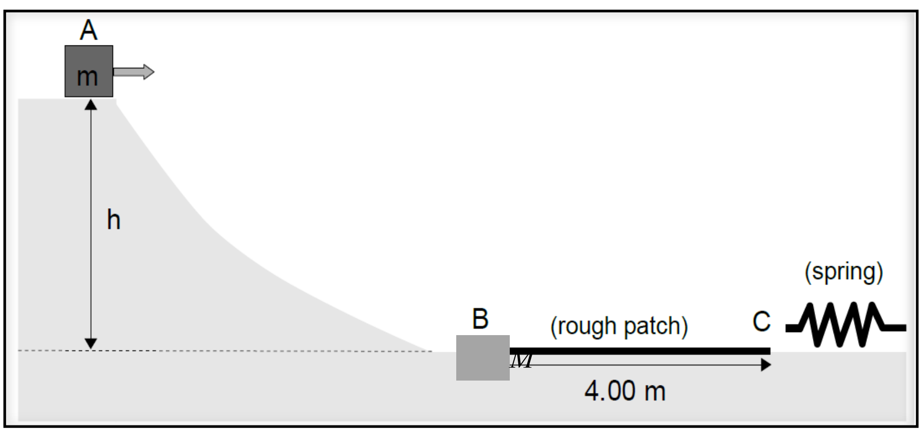
    Part 2: apply the Conservation of Mechanical Energy to the block/ball

    - the final speed from part 1 become the initial speed for part 2

Here is another variation on this plot:

> *A tennis ball, initially at rest, is struck so that it moves straight upward, to a maximum height of 5 m. Calculate the order of magnitude of the average force during the collision. The mass of a tennis ball is about 10 g and the duration of the collision between the racquet and the ball is 10^{-2} s.*

As will be seen in the next section, so-called *expert* problem solvers categorize according to deep structure, what I term theme. Such experts will look at the diagram below and quickly determine possible themes and recognize the bi- or tri-partite plot structure (depending upon the story).[10]



How do experts so categorize, what skills distinguish the expert problem solver, and can these skills be taught? In the next section I present some results from *Physics Education Research* that address these questions.

[10] Diagram by Dr. Celine Lebel, Marianopolis College

3. Problem Solving and Physics Education Research

In short, beginning students perceive problem solving in physics as memorizing, recalling, and manipulating equations to get answers, whereas physicists perceive problem solving as applying a small number of central ideas across a wide range of problem-solving contexts.[11]

> *Leonard et al., Using qualitative problem solving strategies to highlight the role of conceptual knowledge in solving problems*

Expert-Novice Research

The oft-quoted remark that "observations are theory laden" seems to me an apt way to summarize some of the most important results from what has come to be known as "expert-novice" research: experts and novices read the same problem but see something quite different, and this difference is decidedly theory laden—experts perceive deep theoretical structure whereas novices see mostly superficial surface structures. In what follows an expert is taken to be a senior undergraduate or a graduate student in physics while a novice is a student in an introductory general physics course at the undergraduate level.

In their seminal paper, Chi et. al. (1981) investigate differences between the manner in which expert and novice problem solvers categorize and represent physics problems, and the role of such categorization in problem solution. Since the following terms are variously deployed in the literature I will set forth the definitions I intend to use.

- **problem representation**: "a cognitive structure corresponding to a problem, constructed by a solver on the basis of his domain-related knowledge and its organization."(Chi et al. 1981, 122) For Reif a representation is a "redescription of any problem in terms of concepts provided by the knowledge base" (Reif 1979, 1). While other definitions abound all agree that a problem representation will include some or all of the following semantic components:
    - an initial state as set forth in the problem

[11] Leonard et al. 1996, 1496

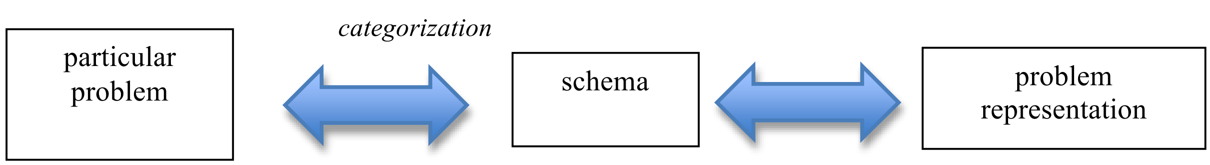
- the goal of the problem-solving operation
- the allowable problem-solving operators
- what are termed embellishments, inferences and abstractions

- **category**: a general problem type
- **schema**: a general problem type consisting of "interrelated sets of knowledge that unify superficially disparate problems by some underlying features" (Chi et al. 1981, 122). In other words, a schema is a category along with its associated knowledge, "the knowledge required to solve routinely a particular class of commonly occurring problems" (Reif 1981, 314). Constructing a representation involves translation from the particular problem at hand to the more universal schema.
- **categorization:** this is the translation process mentioned above.

Schema hails form cognitive psychology and has found its way into cognitive narratology to explain how we construct meaning from narrative:

> The notion of a schema is basic to much of cognitive psychology. A schema is an arrangement of knowledge *already possessed* by a perceiver that is used to predict and classify new sensory data. The assumption underlying this concept is simply that people's knowledge is organized. (Branigan 1992, 13)

The following diagram represents the processes implied by these definitions (really just a species of moving from the particular to the universal), though the actual cognitive sequence is more circular and iterative in nature:



Chi cites various authors whose work demonstrates that the ease and fluency with which experts solve problems and deploy what is termed "intuition" is dependent upon the quality of the representation such solvers are capable of constructing (Hayes and Simon, 1976; Newell and Simon, 1972). Experts quickly provide a tentative categorization, sometimes after reading merely the first phrase, and then elaborate a full representation only after a potential schema has been cued. Following the process of categorization, the schema invokes for the solver an entire associated knowledge base. The assumption is that differences between expert and novice representations lie in their skill (or lack thereof) in problem categorization: "much of expert power lies in the expert's ability to quickly establish correspondence between externally presented events and internal models for these events." (Ibid. 123).

Chi is interested not merely in the existence of schemata—or, I would say, the habit of problem categorization using schema—but rather how such categorization by problem solvers affects their ability to solve problems. The authors report the result of four studies designed to determine the following:

- studies 1 and 2: the categories used (Chi uses "imposed") by experts and novices
- study 3: the knowledge that such categories evoke
- study 4: the features within a problem that cue users as to which schema to use (or, at any rate, use to begin problem solving)

Let us consider these in turn.

Study One: Problem Sorting By Category

Experts (PhD students) and novices (undergraduates) were given 24 problems from standard undergraduate physics textbook, printed on 3"X5" cards, and asked to sort these according to method of solution. They were not allowed a pencil and thus could not actually solve the problem.[12] The results indicate no overall qualitative differences between groups: both groups were able to consistently categorize (they did the exercise twice) into broadly four categories. What *is* interesting—and perhaps unexpected—is that the experts spent considerably more time to categorize (50% longer than novices) though on the second trial the experts, perhaps not unexpectedly, were faster than the novices. So now the important question: if they could both

[12] One wonders if the experts, as with Master chess players, could not see several moves in advance and complete much of the solution in their head, which leads one to wonder if the results of this categorization exercise would be different with more advanced problems.

reliably categorize, on what bases were the categorizations made?

> The most important result form this study is the following: novices sort according to surface characteristics (story) while experts sort according to physical principles (theme).

For example, novices attach great importance to the type of objects in the problem (inclined planes, pulleys, springs), the physical terms mentioned (friction, center of mass) or the relations among objects (block on an inclined plane). Experts, on the other hand, categorize according to "deep structure," the underlying physical principles, the concepts at stake—what I call the theme: Newton's Laws, Conservation of Energy, etc. The differences between expert and novice categorization becomes apparent when reading their brief verbal descriptions for each category, some examples of which are shown below (my table derived from student explanations, Ibid.126-7).

novice	problems that have something rotating: angular speed	blocks on inclined planes with angles
expert	conservation of energy	these can be solved by using Newton's Second Law

Two other conclusions from this study merit mention:

- The sets of expert and novice categories are exclusive—they have little or no overlap with each other. From this the authors conclude that the two groups are using different criteria to sort the problems.
- Experts see and categorize according to unity; novices, diversity. Experts use underlying principles to produce fewer categories where novices tend to see a great variety of different, dissimilar problems. I will address this issue in the conclusion when discussing *conceptual economy*.

The implication for a narrative interpretation of the problem-solving process is quite clear:

experts categorize according to themes; novices, story.

Study Two: Sorting Problems with Surface Similarity

This second study was designed to test one of the conclusions of the first, namely that experts categorize according to deep structure (theme) whereas novices categorize according to surface features (story). For this study problems were chosen that displayed the same surface structure but different deep structure, and the results confirmed expectations based upon the first experiment: experts grouped according to deep structure while novices did indeed group according to surface structure, regardless of differing deep structure. (Of course it is possible that during the solution process some novices *could* have noticed this and been able to correctly determine the deep structure, but this was not part of the study and one can only assume that such a novice would not for long remain a novice.) Intermediate-level problem solvers (upper-level undergraduates) did group by structure, but a grouping constrained by surface features (Ibid. 133), suggesting to the authors an intermediate stage of comprehension and that, most importantly, with learning comes a "gradual release from dependence on the physical characteristics . . ." (Ibid. 134): in other words, they are learning to solve like an expert.

After categorization the problem solver proceeds to the representation stage. A representation is the solver's interpretation or understanding of the problem, "an internal cognitive structure constructed by a problem solver to "stand for" or model a problem" (Ibid. 144-5). Both novices and experts construct what Chi refers to as an "enriched internal representation" of the problem (ibid. 134), although the experts construct more fruitful representations that include idealized objects, concepts and constraints. The authors rely upon the following taxonomy of representations (from McDermott and Larkin, 1978).

- stage 1: a literal representation containing relevant keywords
- stage 2 (*naïve*): a naïve representation (*i.e.* sketch) of literal objects and their spatial relations, termed *naïve* since this can be done by someone lacking the knowledge to solve the problem.
- stage 3 (*scientific* stage): idealized objects (point sources or bodies) and physical concepts (forces, momenta, energies), all of which relate to the method of solution.
- stage 4: an algebraic expression of the problem and its final solution

Obviously stages 2 and 3 are the most important for problem solution. By the time one arrives at stage 4 the problem has been reduced to an exercise. The novice-versus-expert predilections determined in Study One can be recast thus:

Novices categorize problems at the *naïve* level (stage 2)—with some *scientific* elaboration (stage 3)—whereas experts categorize according to similarities at the *scientific* level (stage3).

This difference probably accounts for the longer time taken by experts to categorize: they process problems to a deeper level, but in so doing they have at hand what they believe to be the underlying physical principle, leading—one would assume—to a more timely and satisfactory conclusion of the problem-solving process.

Study Three: Contents of The Schema

What knowledge is accessed by the categories chosen by novice and experts? The authors selected twenty category labels from the expert and novice categorizations in Study One. The subjects were asked to (i) tell everything that they knew about problems from each category and (ii) how the problems might be solved. They were given three minutes for each category. Below are results from two subjects, one expert and one novice, for the category "inclined plane."

A glance at the novice hierarchy below (Ibid, 136) reveals that the initial focus is upon variables such as length and angle, then moving from a consideration of the plane's surface characteristics to Forces and, only at the end, the possibility of Conservation of Energy.

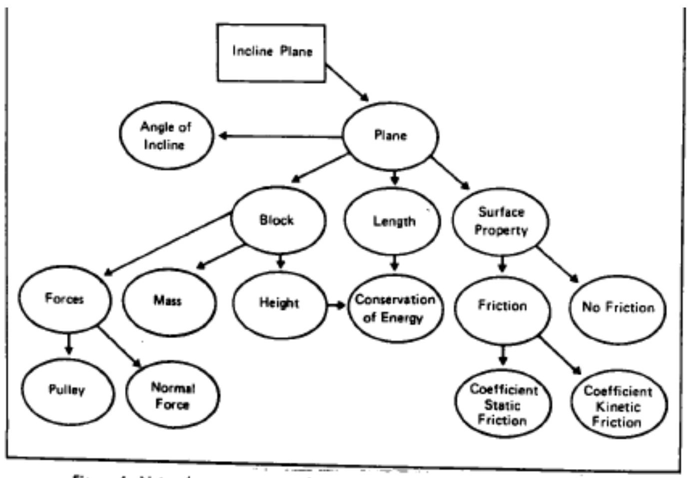
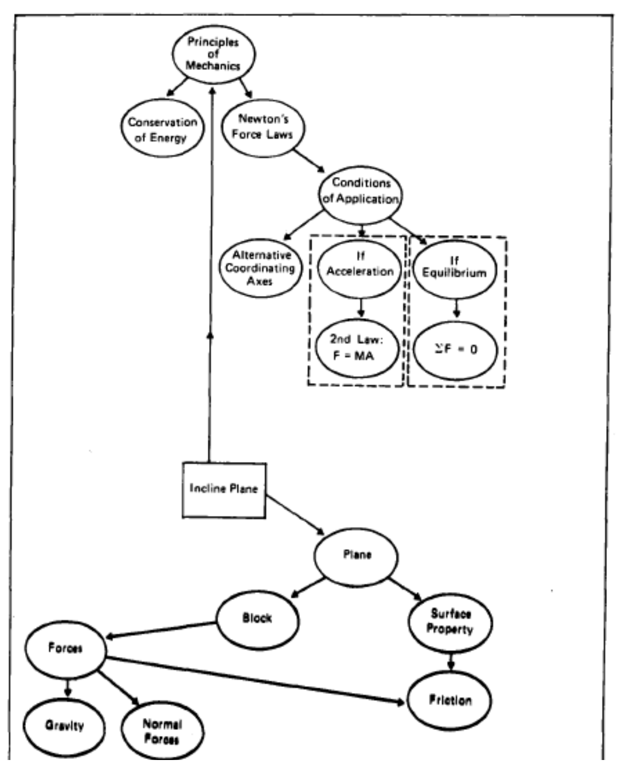


Figure 4. Network representation of Novice H. P.'s schema of an inclined plane.

In contrast the expert network below (Ibid. 137) reveals an immediate concern with physical principles. Two alternatives are mentioned at the beginning, and these both carry with them specific procedural knowledge and conditions of applicability of principles (in the dotted enclosure—basically an incipient solution). Only after identifying possible themes does the expert proceed to elaborate the surface features: these are certainly necessary for problem solution but



do not dictate the expert's elaboration of the representation.

The authors also parsed through responses using "*production rules*" (139), which are basically straightforward translations into sequences of *if-then* statements, what Greeno terms *action schemata* (Greeno, 1980). None of the novice *production rules* contain any actions related to solution procedures whereas the experts' are replete with solution procedures (Chi et al. 1981, 140).

In sum, this study demonstrates that categories invoke for experts a knowledge structure (schema) that includes potential methods of solution, whereas a novice schema invokes a few structural features, many surface features and no methods of solution.

Study Four: Cuing Features

So far we know that experts and novices categorize differently (surface versus deep structure), these categories invoke a schema used to erect and elaborate a problem representation, and this schema (for experts) provide methods of solution. The fourth study attempts to determine what problem features cue subjects' selection among various categories and schema.

The participants took part in a "think aloud" protocol whereby they were asked to read problems and say

1. what basic approach they would take to problem solution, and
2. state what features of the problem led to this choice (cuing features).

Table 12 from the paper shows the first- and second-order features identified by an expert: (Ibid. 144). (The three columns correspond to my theme, plot/motif and story.)

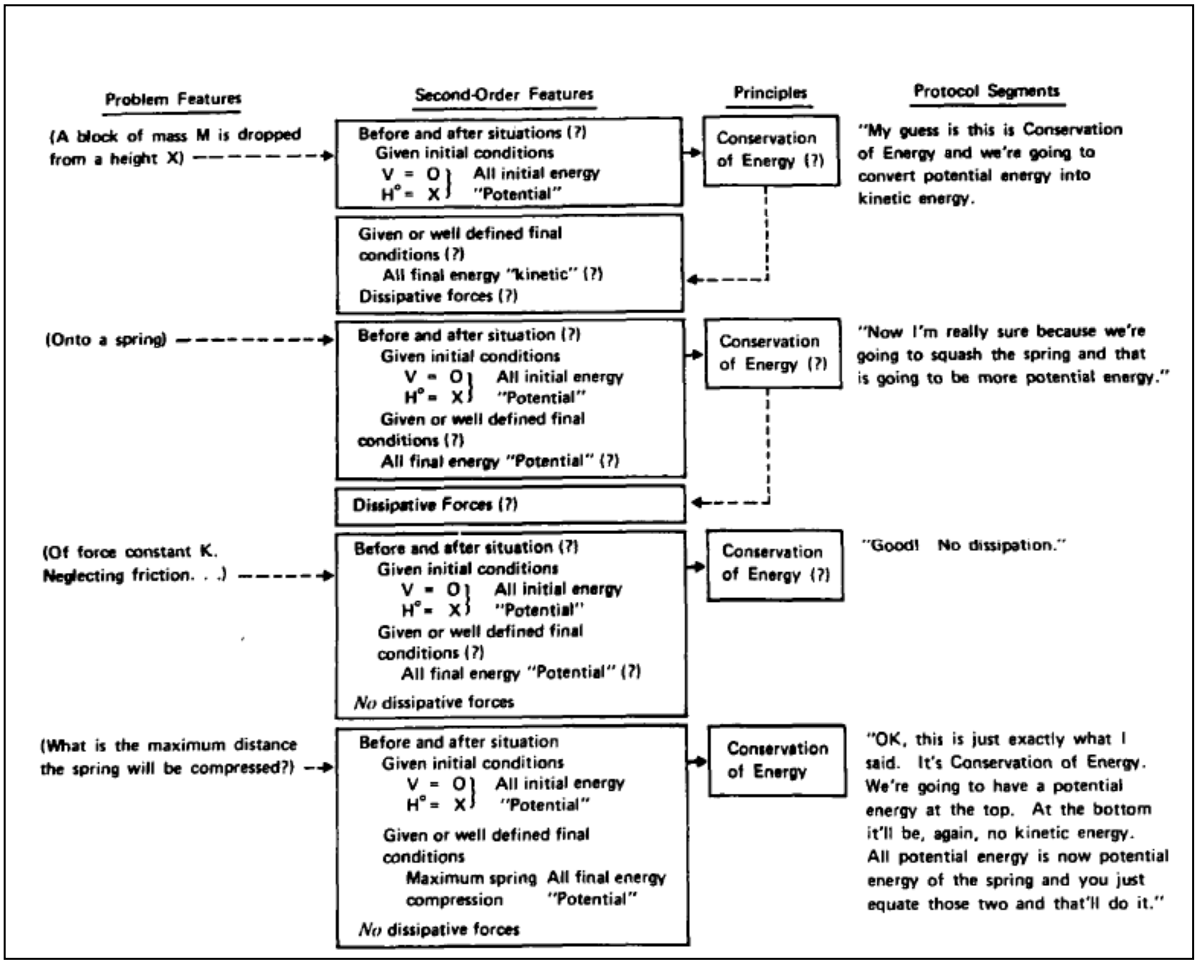
TABLE 12
The First- and Second-order Features that Elicited Expert V. V.'s Final "Basic Approach"

Principles	*Second-order Features (Derived Features)*	*First-order Features (Surface Features)*
Conservation of Momentum (Problems 2, 4, 13)	Before and after situation	Girl on *still* merry-go-round *throws* a rock . . . Two initially separated wheels are *suddenly coupled*.
	No external forces	Neglecting friction. No third entity mentioned except the interacting wheels.
Conservation of energy (Problems 5, 7, 16, 19, 20)	Before and after situation	Block *dropped* from a height X *onto* a spring. Block *starts* with initial velocity V. How *far* will it slide?
	Given or well-defined initial conditions	Initial height = X. Initial velocity = 0. Initial velocity = V.
Force Laws (Problems 3, 8, 9, 11, 12, 14, 15)	Determination of something at an instant in time	Break point of a rope. Coin observed to slide at distance R from center of turntable. Raising point of a disk.

My table below summarizes the findings from this study. (Ibid. 142)

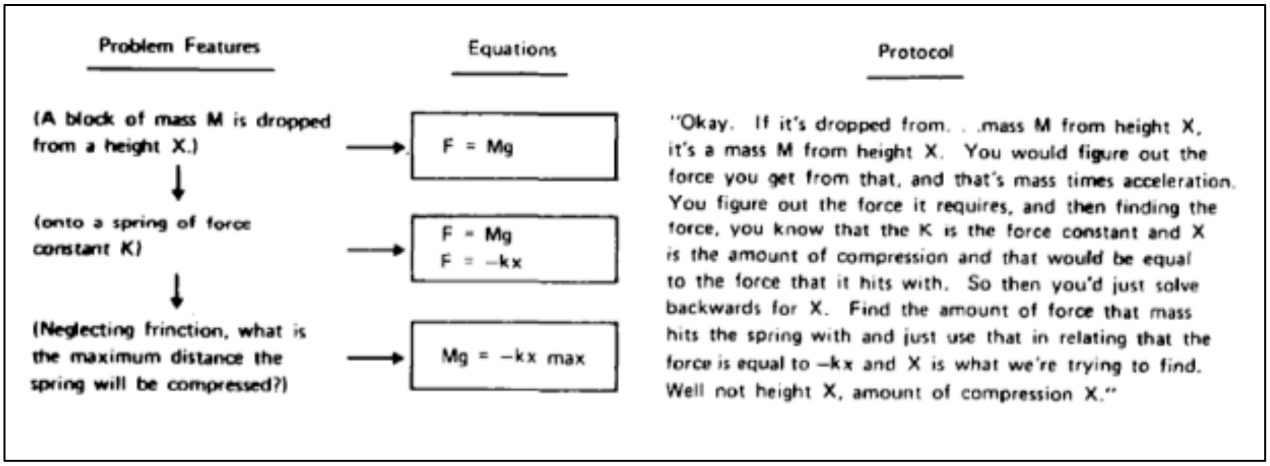
	basic approach	**cuing features**
expert	▪ interpreted "basic approach" to mean the physical principles used to solve a problem ▪ used the same terms to describe their basic solution approach as experts used in problem categorization ▪ near perfect agreement among all experts (not surprising since they are working at the thematic level)	▪ states and conditions of the physical situation not contained in the problem statement, hence termed second order ○ before and after ○ no external forces
novice	▪ produced only vague procedural statements (such as "first figure out what's happening") or gave detailed equations to solve the problem	▪ literal terms from the problem itself (hence termed first order) that give rise to equations ▪ gravity, friction ▪ second order almost never used by novices ▪ no overlap in features mentioned by two groups ▪

A further analysis of the think aloud data investigates the subjects' protocols in constructing their problem representation. The following chart takes us in time through an expert reading of a problem. (Ibid. 146)



The first column contains literal segments from the problem and column 4 the thoughts of the expert after reading thus far in the problem. The second and third columns identify the second-order features and principles the subject deduces from the problem segment. After reading the first phrase ("A block of mass M is dropped from a height x . . ." the expert has activated the "Conservation of Energy" schema (theme) and this in turn generates what the authors refer to as *slots* that guide the reader in establishing or rejecting this schema. Examples of slots (what I would call plot/motifs) are "before and after situations; given initial conditions." The expert continues to read and translate literal features into first- and second-order problem features, all the while confirming the initial hypothesis of Conservation of Energy (expert selection of the principle is guided by second-order derived knowledge.). The final expert protocol includes a form of the equation that will no doubt be used in the solution.

The novice, by contrast, activates schema based on first-order literal cues: "dropping" cues gravity; "spring" cues Forces, etc. (chart below, Ibid. 148). The slots deal mainly with equations, and as can be seen in the example below the novice reliance on surface level (first-order) categorization led to inappropriate equations associated with the surface level only.



In sum, this last study demonstrates that expert selection of physical principle is based on second-order, derived knowledge; experts are cued not by words but rather by the signification of the words. Novices—always in contradistinction to experts—used literal first-order cues that could lead to an equation.

Expert, Novice And Narrative

One significant difference between experts and novices is the "structure" of their knowledge: experts have a large store of domain-specific knowledge, and this knowledge is richly interconnected and interrelated—it is organized to be useful. Novices, by contrast, have a sparse knowledge store with gaps, forming disconnected groupings around distinct topics. Experts structure their knowledge hierarchically, organized by fundamental principles; novices store theirs chronologically as it is learned. (Gerace and Beatty 2005, 3) Experts integrate multiple representations of ideas, whereas novices often have only one representation, or are unable to relate different representations. As a result, experts have good recall of their knowledge, and can access whatever part is relevant for a problem, while novices have poor recall and don't have access to a particular bit unless it is somehow cued—perhaps by a familiar, standard problem type that they have been drilled to associate it with.

Expert-novice distinctions tend to be of the binary sort:

> Experts and novices . . . differ in their problem solving behaviors. Experts employ forward-looking concept-based strategies, whereas novices typically employ backward-looking means-ends techniques . . . Experts can think about and monitor their problem solving while engaged in it, but for novices, problem solving consumes all available mental resources. Finally, experts can and generally do check their answers via alternative methods, while novices usually have only one way to solve a problem. (Ibid. 3-4).

In this extensive summary one recognizes in the expert habits many of the cognitive gains and efficiencies promised in the *narrative effect* discussed in the Introduction. Of course experts have not become so due to their diligent literary studies, or have they? In his article *How Stories Make Us Smarter*, David Hermann discusses how stories "provide important representational tools for humans—tools that facilitate a number of problem-solving activities." (Herman 2003, 135) Hermann identifies five problem-solving activities that are supported by narrative tools, two of which are germane with respect to expert-novice studies.

1. "chunking" experience into workable segments

> The knowledge of experts consists of a large number of interconnected elements that are stored and recalled as extended, coherent chunks of information organized around underlying principles in the domain. Experts use the structure of this knowledge to perceive and recognize underlying patterns and principles in problem situations . . . (Russel 1997, 950)

> . . . narrative affords representational tools for addressing the problem of how to chunk the ongoing stream of experience into bounded, cognizable, and thus usable structures. Stories organize experience by enabling people to select from among the total set of sequentially and concurrently available inputs. (Herman, 2003, 136)

It is easy to recognize in these quotes the expert's ability to categorize by problem type (by theme and plot)—the thousands of end-of-chapter problems become a handful of themes and plots.

2. imputing causal relations between events

> Narrative can be construed as both reflecting and supporting a cognitive

> predisposition to find causal links between entities, states, and events in a sequentially presented array. (Ibid. 137)

Causal links between initial and final states, or between entities, is the very stuff of physics problem solution, and experts find such links at the thematic level, following second-order cues that give rise to equations, states and conditions of the physical situation not contained in the problem statement. Novices ignore possible causal links and rely instead upon literal terms from the problem itself (first-order cuing features) to activate an algebraic manipulation, working backward from the unknown quantity, "a general heuristic called means-end analysis, which is typically described as 'plug and chug'." (Maloney 2011, 10).

getting unstuck

It is fascinating to watch good students find their way out of a problem by probing by scribbling, sketching and adjusting their approach in response to derived knowledge (Study Four above). In contrast the novice reliance upon first-order cues means that, if stuck, they have exhausted their only resource and give up: "Experts have a variety of tactics for getting unstuck, but novices cannot generally get unstuck without outside help." (Gerrace and Beatty 2005, 3-4) During the problem solution process one begins with a certain representation and works on that representation (and hence the choice of representation, or at any rate the ability to produce and elaborate a representation, is of the greatest import). But one must be ready to adjust to any problems or new information that challenge the initial interpretation. This iterative approach to problem solving is illustrated by expert think aloud protocols: experts reevaluate in light of new, derived second-order knowledge (knowledge which novices could not generate for themselves).

The findings from cognitive narratology are similar: "unexpected information can cause a reorientation of the schema in order to reclaim the important from the superficial." (Branigan 1992, 16) Readers, like expert problem solvers, do not parse a narrative in a linear fashion:

> In short, it has been amply demonstrated through many psychological experiments that an individual's attention does not spread equally through a narrative text but works forward and backward in an uneven manner in constructing large-scale, hierarchical patterns which represent a particular story

> as an abstract grouping of knowledge based on an underlying schema. (Ibid.)

And it is not merely a matter of parsing in both directions but of invoking one's schema to generate and test expectations:

> . . . story comprehension involves the continuous generation of better-specified and more complicated expectations about what might be coming next and its place in a pattern. Thus the perceiver will strive to create "logical" connections among data in order to match the general categories of the schema. . . the "gist" of the narrative . . . uses a schema to automatically fill in any data that is deemed to be "missing" in the text. (Ibid. 15-16)

From Novice To Expert: Problem-Solving Strategies

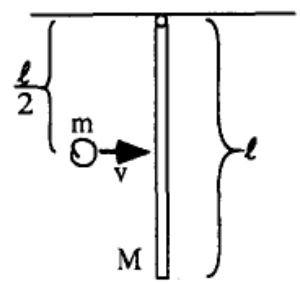
As one author put it, ". . . relatively inexperienced physics students tend to plunge immediately into equations and, lacking any guiding plan, often get stuck in a morass of details without knowing what to do next."(Reif, 312) One strategy (pun intended) for helping such students bridge the gap between novice and expert habits is to require that they articulate a solution strategy. A *strategy* is defined as a qualitative description consisting of three components, namely the *what*, *why* and *how* (which three terms certainly call to mind the climactic scene of murder mystery):

- state *what* major physical concepts or principles can be used to solve the problem
- justify (articulate) *why* these principle can be used
- describe *how* the principle or concept is used to arrive at a solution

Below is an example of a problem with a strategy (Leonard et al. 1996, 1497):

Problem:

A stick of mass M and length ℓ is hanging from a ceiling as shown to the right. A piece of putty of mass m is traveling horizontally when it strikes and adheres to the stick at its midpoint. What must have been the speed of the putty, v, if the stick (with putty attached) swings and comes momentarily to rest in a horizontal position before starting to swing down again?



Strategy:

This problem consists of two parts. First, the mass and stick undergo a totally inelastic collision. Angular momentum is conserved about the pivot during the collision since there is no net external torque (about the pivot). Even though there are two external forces present during the collision, these do not provide a net external torque: The force at the pivot has no moment arm (i.e., $r = 0$) and so it does not provide a torque about the pivot. The gravitational force acting on the center of mass of the stick points vertically down and initially is parallel to **r** (i.e., angle between **r** and $\mathbf{F_g}$ is 0°). Assuming the collision is instantaneous, the gravitational force delivers no angular impulse to the system <u>during the collision</u>. So, assuming there is no friction between the pivot and the rod, there are no angular impulses delivered to the putty–stick system. Thus, for the collision portion of the problem, equate the initial and the final angular momenta of the putty–stick system. Note that only the putty contributes to the initial angular momentum since the stick is at rest prior to the collision.

The second part of the problem consists of the putty–stick system rotating to a horizontal position before momentarily stopping. During this portion of the problem, mechanical energy is conserved. Only two forces act on the system during this "swinging" portion: The gravitational force and the force at the pivot. The force at the pivot does no work since the pivot point does not move and we have assumed that there is no friction between the pivot and the stick. Only the gravitational force (a conservative force) does work, resulting in mechanical energy being conserved. Thus, for the second portion of the problem, equate the initial and the final mechanical energies of the system. Assuming the collision was instantaneous, the initial orientation of the putty–stick system is vertical. The center of mass of the system is at the midpoint of the stick. (We need this to evaluate the potential energy of the system.) Applying the two concepts of angular momentum and energy conservation will result in two equations that will allow you to solve for v in terms of the "givens" in the problem.

For the authors *strategies* are a means of integrating conceptual knowledge with problem solving, and also provide opportunities to model for students "the type of concept-based, qualitative reasoning that is valued in our profession." (Ibid. 1495) The problem that the authors address in this paper is a familiar one:

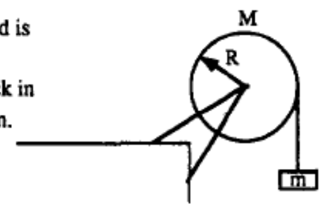
> . . . the way we model problem solving for students does little to alter their predisposition to focus on finding and manipulating equations . . . although we are usually careful to state verbally the principle or concept being applied to solve a problem, we often only *write down* the equations by which the principle is instantiated. . . (Ibid. 1496)

So in fact our modeling reinforces student perception that "problem solving involves manipulation and that principles are abstractions that bear little relevance to obtaining answers to problems.” (Ibid.)

Below are examples of what the authors term A- and B-level strategies that could just as well be labeled expert and novice.

Problem:

A disk of mass, M = 2 kg, and radius, R = 0.4 m, has string wound around it and is free to rotate about an axle through its center. A block of mass, m = 1 kg, is attached to the end of the string and the system is released from rest with no slack in the string. What is the speed of the block after it has fallen a distance, d = 0.5 m. Don't forget to provide both a strategy and a solution.



Above Average Strategies:

A1: Use the Conservation of Energy since the only non-conservative force in the system is the tension in the rope attached to the mass M and wound around the disk, (assuming there is no friction between the axle and the disk, and the mass M and the air), and the work done by the tension to the disk and the mass cancell each other out. First set up a coord. syst. so the potential energy of the system at the start can be determined. There will be no kinetic energy at the start since it starts at rest. Therefore the potential energy is all the initial energy. Now set the Initial energy equal to the final energy that is made up of the kinetic energy of the disk plus the mass m and any potential energy left in the system with respect to the choosen coord. system.

A2: I would use conservation of mechanical energy to solve this problem. The mass m has some potential energy while it is hanging there. When the block starts to accelerate downwards the potential energy is transformed into rotational Kinetic Energy of the disk and Kinetic energy of the falling mass. Equating the initial and final states; and using the relationship between v and ω the speed of m can be found. Mechanical energy is conserved even with the non-conservative Tension force because the Tension force is internal to the system. (pulley, mass, rope)

A3: The linear velocity is related to the angular velocity by the factor of the disc's radius. In this problem the pivot is frictionless so all energy is conserved. By equating the initial energy (potential of hanging mass) with the final energy (kinetic of mass & kinetic of rotating disc), it is then possible, after changing the rotational velocity to its linear form, to isolate and calculate a value for the final linear velocity.

A4: Apply Newton Second Law to find the net force acting on the block. This is equal to its mass times it's acceleration. This net force, crossed with the radius of the disk, provides a net torque. The net torque is the moment of inertia of the disk times the angular acceleration. Solve both equations for the accelerations. They are related because the acceleration is the cross product of the radius and the angular acceleration. Substitute what you solved for in the 2 equations above, and then solve for Tension (T). Substitute this for T in the equation for acceleration and then solve using math.

Below Average Strategies:

B1: Using d=0.5 m, we could find θ and from there we could figure out the time using θ, α, and ω_0. After we found this, we could use that, along with the other given information to determine the angular speed. Once we know this, we can relate the angular information to the block.

B2: Not only do you have to consider the mass of the block on the string but also the force of gravity on the block. Rotational kinematics must be used with the radius, mass and gravity.

B3: In trying to find the speed of the block I would try to find angular momentum kinetic energy, use gravity, I would also use rotational kinematics and Moment of Inertia around the center of mass for the disk.

B4: There will be a torque about the center of mass due to the weight of the block, m. The force pulling downward is mg. The moment of inertia about the axle is 1/2 MR^2. The moment of inertia multiplied by the angular acceleration. By plugging these values into a kinematic expression, the angular speed can be calculated. Then, the angular speed times the radius gives you the velocity of the block.

The A-level strategies display all three components of a strategy (what, why and how), focus on thematic material (the physical principles), were well written and in general correspond with our expectations vis-à-vis expert-novice studies. The B-level strategies also correspond well with expert-novice expectation, betraying a focus on literal features—such as the variables displayed in the drawing and the geometry—along with some rudimentary concepts of motion and a disposition to move directly toward algebraic manipulation to find the unknown. More importantly, one notices in the B-level strategy a dearth of major physical principles.

To test the efficacy of strategy writing the authors studied two classes, one control class taught in the traditional fashion and the other taught with an emphasis on strategies—that is, integrating conceptual knowledge with problem solving. Those taught in the usual lecture style did not display any movement from novice to expert habits: "The control group continued to display novice habits and did best on questions where superficial features happened to match solution principle." (Ibid. 1500)

By contrast the results for the other class were encouraging: students taught with strategies selected the appropriate principle 50% more often than the control group, were found to be "less dependent on the surface features alone for selecting an appropriate principle" (Ibid. 1501) and demonstrated a greater recall of important physical concepts six to eleven months after the course. Thus the authors claim that novice students can indeed gain an appreciation for the value of thinking in terms of physical principles in solving problems.

> . . . a modest effort to focus attention on the role that physics principles play in solving problems can help students retain the major ideas presented in a physics course months afterward." (Ibid. 1502)

Such a program requires effort on behalf of the teacher and student: separating strategy from solutions is not a simple task for many students, certainly one they would rather continue to avoid in favour of algebraic manipulation. But given that the authors marked strategies on the exam questions—and made it worth 13% of the grade for a given problem—students took the task seriously. (Ibid. 1498)

Strategy writing appears to be a successful method for helping students to organize their knowledge and problem-solving tactics (see also Gick 1986; Widmayer 2005), yet I find the exclusive reliance upon written language to be cumbersome and limiting. While scientific text *does* use written language it also relies heavily upon other modalities for communication and for what is termed *meaning making*. In the next section I survey the semiotics[13] of scientific communication and propose my own narrative-based templates for problem solutions.

[13] Briefly, *semiotics* is the study of signs and the meanings produced by various sign systems (either semantic or other formal structures).

4. The Problem Solution: Constructing and Transmitting Meaning

Experts and successful novices spent time using the external representations to make sense of the physics in the task, while unsuccessful novices seemed to draw pictures and free-body diagrams out of a sense of requirement[14].

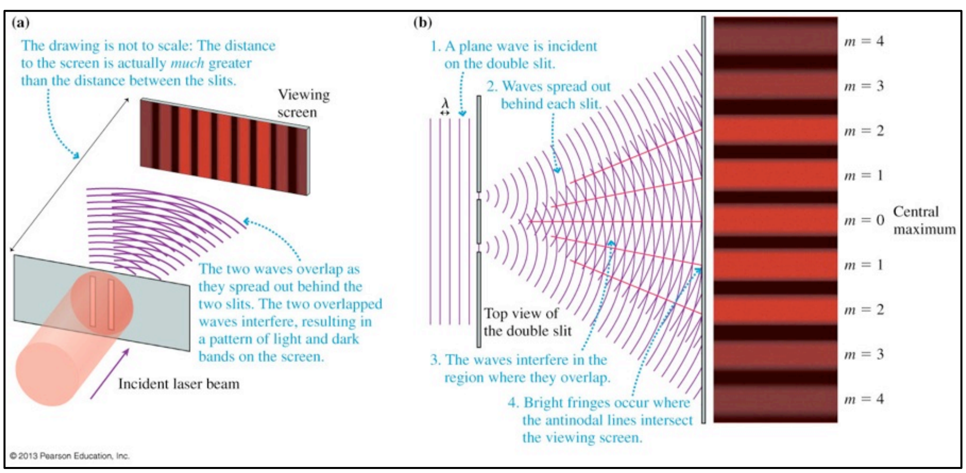
David Maloney An Overview of Physics Education Research on Problem Solving

The Language Of Science

the language of science is multimodal

The language of science is multimodal: we use graphs, tables, diagrams, equations and gestures to communicate meaning. Consider, for example, the phenomena of wave interference produced by passing light through a double-slit apparatus. To construct a problem representation one begins with a figure representing the actual physical apparatus (story) and then continues through a series of reductions designed to represent the thematic and motivic content: the following sequence can be found in almost any textbook presentation of the subject:

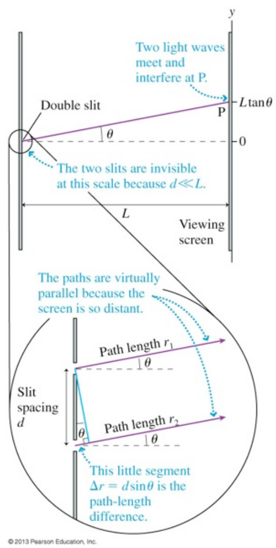
1. A figure or sketch representing the physical situation (story level, figure (a) below).



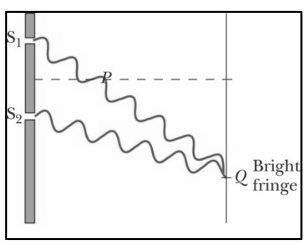
2. A reduction of the physical apparatus to a schematic (figure (b), both from Knight 2012, 629).

[14] Maloney 2011, 15.

3. A further reduction to reveal geometry and any applicable approximations (in this case the path difference for light traveling from the two slits and meeting at some point on the screen is found to be $\Delta r \approx d\sin\theta$ d<<L). (Ibid. 630)

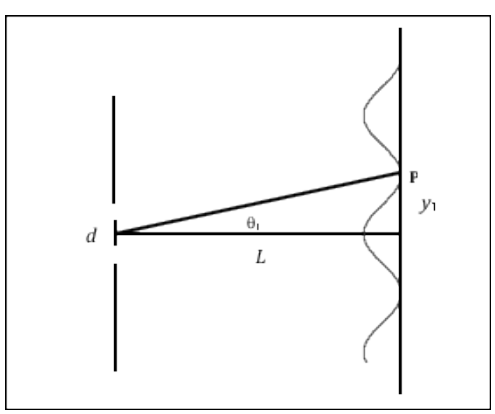


4. Possibly a reduction illustrating wave interference at a point. (Serway 2004, 1179)



At this point a statement of physical principle (rather than a geometric approximation) is appropriate: $\Delta r = m\lambda$, for example, as the condition for constructive interference at point Q.

5. A final schematic that allows for problem solution.



This diagram may also include other relevant variables such as the slit widths and the wavelength of the source. Also included are the point *P* at which we are interested in the state of affairs (namely, is the interference constructive or destructive at that point) and some method of representing the interference pattern seen (or, at any rate, measured) on the screen. (The resulting interference pattern can be represented by a quick sketch of the intensity pattern, as it is above, or by using shaded and non-shaded regions, or merely alternating letters **B** and **D** to locate the bright and dark fringes.)

What follows or accompanies the diagram(s) is a statement of the physical condition for either constructive or destructive interference: for destructive interference at the point P the path difference must satisfy the condition

$$\Delta r = (2m+1)\frac{\lambda}{2}$$

where the geometric expression for this path difference is given by

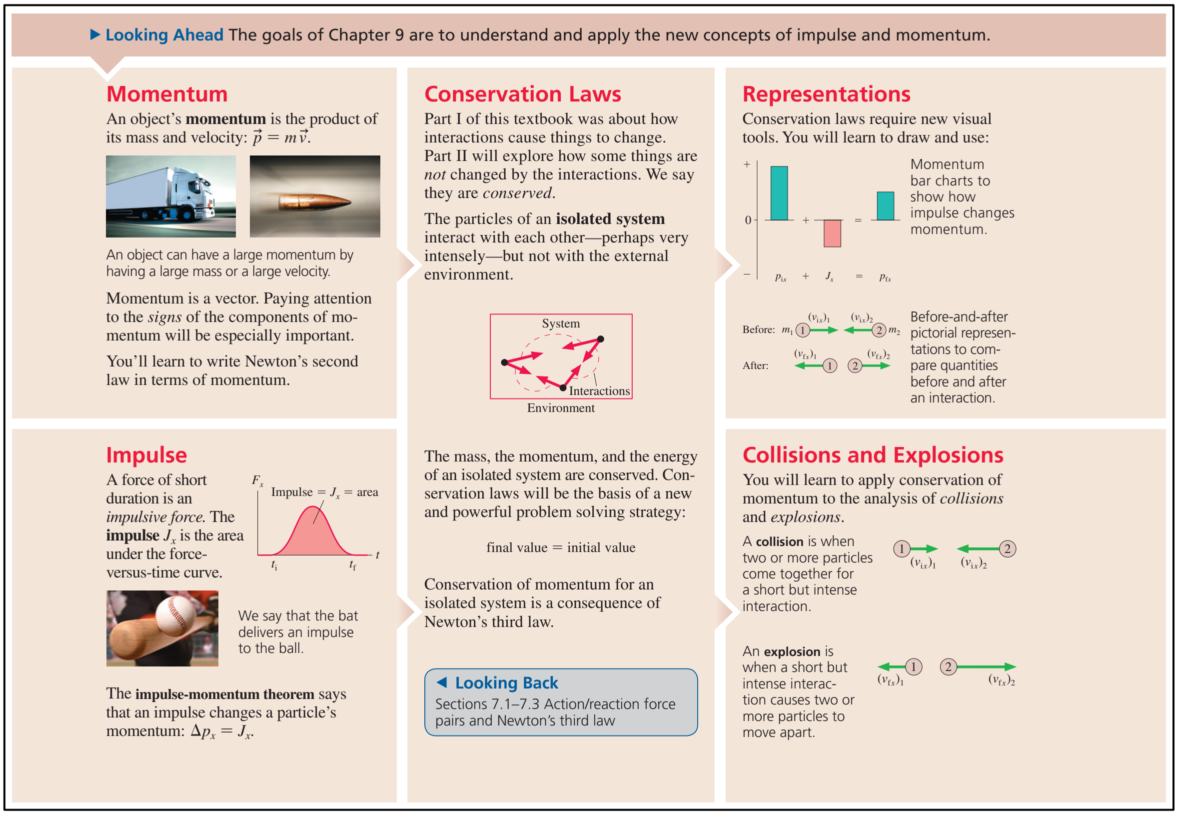
$$\Delta r \approx d \sin\theta \, .$$

At this point one may proceed with an algebraic solution to many a problem.

In textbook materials, articles or problem solutions information and concepts are communicated using pictorial, graphical and mathematical representations—the language of science is multimodal:

> The concepts of science . . . are semiotic *hybrids*, simultaneously and essentially verbal, mathematical, visual-graphical, and actional-operational . . . To do science, to talk science, to read and write science it is necessary to juggle and combine in various canonical ways verbal discourse, mathematical expression, graphical-visual representation, and motor operations in the world. (Lemke 1998, 87)

Students are at some level aware of the various multimodal meaning-making devices: lectures and textbooks offer frequent exposure to multimodal resources (see the highly effective chapter summary below (Knight 2012, 220)).



meaning is transmitted by more than one modality

The various modalities used are no mere shorthand for written language. Writing evolved with other modes of communication and what J. L. Lemke terms "meaning making:"

> In its efforts to describe the material interactions of people and things, natural science has been led away from an exclusive reliance on verbal language. It has tried to find ways to describe continuous change and co-variation, in addition to categorical difference and co-distribution. It has tried to describe what we know through our perceptual Gestalts and motor activities, to construct representations of the topological as well as the typological aspects of our being-in-the-world. Language, as a typologically oriented semiotic resource is unsurpassed as a tool for the formulation of difference and relationship, for the making of categorical distinctions. It is much poorer (though hardly bankrupt) in resources for formulating degree, quantity, gradation, continuous change, continuous co-variation, non-integer ratio, varying proportionality, complex topological relations of relative nearness or connectedness, the interpenetration of different dimensionalities, or nonlinear relationships and dynamical emergence. (Ibid. 87)

The vocabulary of science and the related concepts are simply not suited to verbal language:

> We can indicate modulation of speed or size, or complex relations of shape or relative position, far better with a gesture than we can with words, and we can let

that gesture leave a trace and become a visual-graphical representation that will sit still and let us re-examine it at our leisure. (Ibid.)

Lemke surveys scientific articles and textbooks, detailing the frequency of non-verbal semiotic means such as graphs, tables, figures and equations. He finds that without the other modes the verbal text would make no sense, and that the content of the visual information cannot simply be replaced by verbal language:

> In most of the theoretical physics articles, the running verbal text would make no sense without the integrated mathematical equations, which could not in most cases be effectively paraphrased in natural language, even though they can be, and are normally meant to be read as if part of the verbal text (in terms of semantics, cohesion, and frequently grammar). (Ibid. 89)

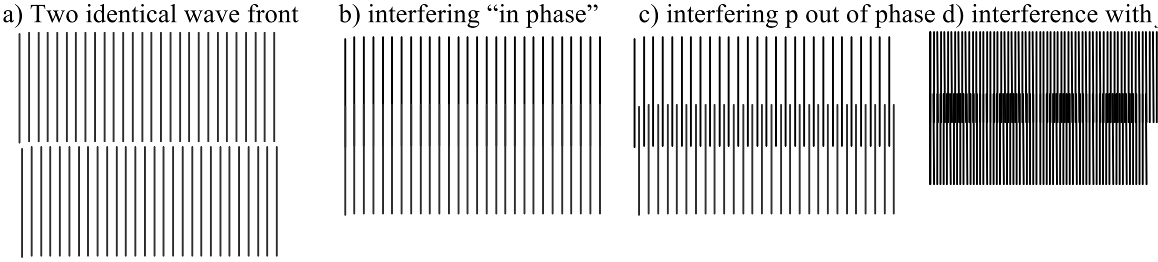
The paper from which this quote is taken is entitled *Multiplying Meaning*, by which title the author conveys the essence of his semiotic analysis: science requires several simultaneous forms of communication, these various forms contain information—that is, *meaning*—that could not otherwise be delivered, and the interaction among the various forms leads to a sum which is much greater than its parts:

> In multimedia genres, meanings made with each functional resource in each semiotic modality can modulate meanings of each kind in each other semiotic modality, *thus multiplying the set of possible meanings* [my italics] that can be made . . . (Ibid. 92)

Consider the multiple representations used during a lecture presentation of acoustic beats.

1. The phenomenon is demonstrated, usually using two amplified tuning forks.
2. A visual analogy: Moiré Lines

a) Two identical wave front b) interfering "in phase" c) interfering p out of phase d) interference with f_2=$0.9^{ish}f_1$



3. The equation for the superposition of two waves

$$\begin{aligned} y(t) &= y_1(t) + y_2(t) \\ &= A\cos(\omega_1 t) + A\cos(\omega_2 t) \end{aligned}$$

At this point I ask students if the above equations in any way resemble what was heard during the demonstration.

4. Application of a trigonometric identity simplifies the above expression:

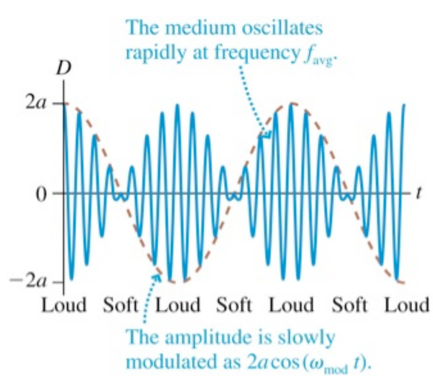
$$y(t) = 2A\cos\left(2\pi f' t\right)\cos\left(2\pi \bar{f} t\right)$$

where $\begin{aligned} f' &= \left(f_1 - f_2\right)/2 \\ \bar{f} &= \left(f_1 + f_2\right)/2 \end{aligned}$

I repeat the above question: Does this look like it could possibly represent was heard?

5. The two terms of $y(t) = 2A\cos\left(2\pi f' t\right)\cos\left(2\pi \bar{f} t\right)$ are graphed separately (using the frequencies of the tuning forks from demonstration—eg 284 Hz and 286 Hz), the graphs stacked one above the other to show a slowly-varying sinusoidal function (the envelope function with period $\bar{f}$, perceived in the demonstration as the alternation of loud and soft) and a rapidly varying function with period f' (corresponding to the average frequency, the pitch perceived in the demonstration).

6. Beneath these curves the graph of their product is shown (Knight 2012, 619):



Again I ask the students if our new representation resembles what was heard, and by this point the answer for many is yes.

For attentive students—it is hoped—the interaction of the modalities in this sequenced presentation enhances understanding and learning in a way that a separate consideration of each could not:

> . . . I analyzed how science teachers and students made sense with each other by co-deploying verbal, gestural, and pictorial resources. I found that if we regard each of these as constituting a separate "channel" of communication, then sometimes the same or equivalent information passes nearly simultaneously in more than one channel, sometimes the information in the two channels is complementary, and sometimes information comes first in one channel, and later in another. It became very clear to me that the meanings that were being constructed were joint meanings produced in the intersection of different semiotic systems. While it was useful to analytically separate these into different "channels," there was also an underlying unity to the meanings produced. Their separation neglects this fundamental unity of communicative meaning-making which makes the co-ordination among channels not only possible, but normal. (Ibid. 94-5.)

It is this coordination between modalities that may often be absent from the lecture hall, but most certainly from students' written solutions. Before writing the solution, however, the problem must be solved.

Constructing Meaning: Comprehension And Problem Solution At The Murder Board

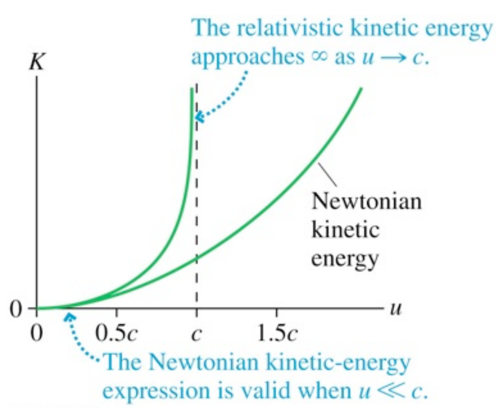
The coordination of modalities Lemke refers to in the previous quote is, I believe, crucial for the problem-solving process, for to achieve the clarity which is the solution one must construct meaning for oneself, and, as we have seen, meanings and concepts in science are multimodal. The expert habit that I would like to introduce to my students is the multimodal reasoning and exploration that occurs at the whiteboard, or on a piece of scrap paper, the back of an envelope, whatever. It is here that the real work of problem solution occurs, for to transmit meaning one must first construct that meaning. I know of no obvious algorithm to teach students how to proceed at the board: no doubt there is some iterative process of reading the questions, passing through various reductive diagrams (from story level to deeper plot and thematic structure), deriving second-order knowledge to guide their selection of theme, writing equations and gesturing:

> When scientists think, talk, work, and teach they do not just use words; they gesture and move in imaginary visual spaces defined by graphical representations and simulations, which in turn have mathematical expressions that can also be integrated into speech. When scientists communicate in print they do not produce linear verbal text; they do not even limit their visual forms to the typographical. They do not present and organize information only verbally; they do not construct logical arguments in purely verbal form. They combine, interconnect, and integrate verbal text with mathematical expressions, quantitative graphs, information tables, abstract diagrams, maps, drawings, photographs, and a host of unique specialized visual genres seen nowhere else. (Ibid. 88)

Consider a few of these "specialized visual genres" that are available for use in problem solution.

Graphs

When used in problem solutions graphs most often depict conceptual relations (linear or inverse variation, etc.) rather than actual data. The visual semiotic of graphs allows one to see functional dependencies: "we apprehend the 'patterns' in the data when displayed as a graph differently than we do when it is displayed as text, or even as a Table." (Ibid. 102) The following graph contrasts the asymptotic versus quadratic behaviors of kinetic energy in Newtonian and Relativistic physics: (Knight 2012, 1091).



For students well versed in graphical techniques there is little need for textual explication of the graphs—the striking conflict between classical and relativistic theory (and experiment) is quite evident.

Diagrams

> A diagram is a *visual sign* (known more technically as an *icon*) that represents information concretely, and thus much more comprehensively, by highlighting or showing what the essential features of the problem are, generally in outline form.” (Danesi 2008, 4)

The ability to perform successive reductions of diagrams is a critical competency for successful problem solution, one that involves deciphering correspondences between sets of signs and then representing them in a manner that allows one to determine solution strategies (see Danesi 2008, 10). (For example, as used in the double-slit example, or free-body diagrams in Mechanics.)

Diagrams and graphs can be combined to convey a wealth of information. The figure below (Figure 5.3, Lemke 1998, 103)—taken from a textbook on turbulence—is not described mathematically or textually (*e.g.* “temperature rises linearly with depth of fluid”), there is simply no need: a competent reader does not require such a verbal sequence.

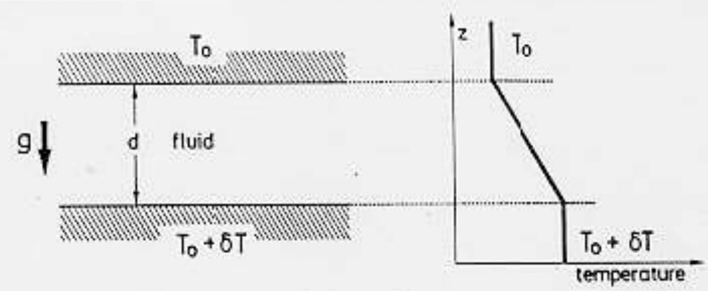


Figure V.3 Diagram of a Rayleigh-Bénard experiment.
The fluid between two horizontal heat-conducting plates has — in the absence of convection — a constant temperature gradient (represented on the right). Its temperature goes from $T_0 + \delta T$ at the bottom of the layer, to T_0 at the top.

The horizontal dotted line integrates a graph with a diagram by means of common variable *z,* thereby linking a spatial variable with the visual representation. The relationship between temperature and position is represented as though it were a shape in space, something Lemke refers to as a “visual metaphor.” (Ibid. 104)

Tables

It may seem that we rarely use tables in problem solution, but consider the following rather straightforward exercise and the method used to organize the information:

Find the energy released in the following nuclear reaction. (121.0 MeV)

$$ {}^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \longrightarrow {}^{141}_{56}\mathrm{Ba} + {}^{92}_{35}\mathrm{K}r + 3\left({}^{1}_{0}\mathrm{n}\right) $$

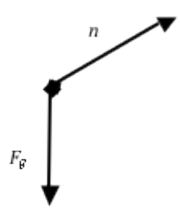
$\sum m_i$	$\sum m_i$

The net mass of both reactants and products are displayed (and calculated by students) in essential tabular form.

examples: constructing meaning with various modalities

Example 1. *free-body diagrams*

The quintessential representation for dynamics problems, the free-body diagram allows one to isolate each body in a problem and consider all forces acting on it without the distraction of other interacting objects (each of which will have diagrams of their own). For me a free-body diagram can and should reveal something of the plot structure. Consider the following novice diagram:

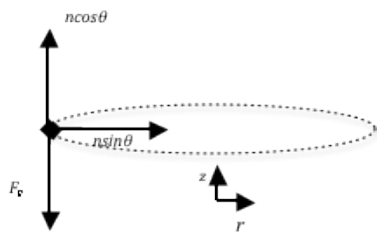


Is this the free-body diagram for a mass on an inclined plane or possibly a car turning on a banked road? To clarify the situation one can use any or all of the following semiotic devices:

- sketch the direction of the acceleration
- draw the coordinate axes that will be used
- resolve the vectors into components along those axes

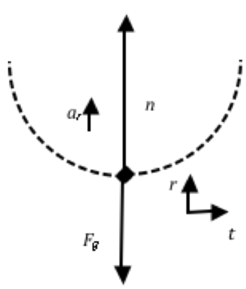
- sketch the circular path (in the case of rotational motion)

Thus the final free-body diagram for a car travelling in a circular path on a road banked at an angle θ:



The following diagram presents a violent clash with most students' intuition. Why is *n>mg?*

Addition of several semiotics clarifies the situation: the body is moving in a circular path and thus there is a net radial force of magnitude *n-mg*.



(Of course one must be careful not to make the diagram too busy—clarity is a virtue).

From this last example we also see that one can convey physical principles (make meaning) by the relative size of the force vectors. In this example $n > mg$ because the net force must be radial for an object moving in uniform circular motion. Requiring students to sketch the approximate size of their vectors (and modeling this for them) forces an intellectual engagement beyond novice habits.

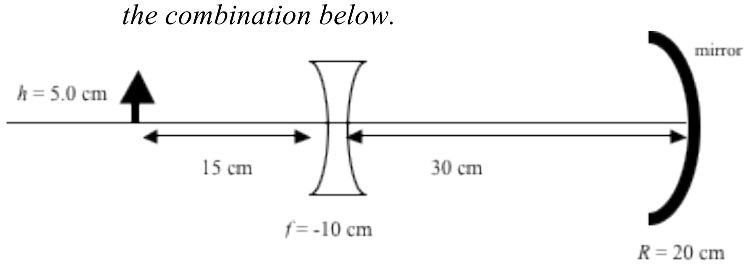
When two or more bodies are involved important thematic content (such as Newton's Third

Law) can be conveyed by certain notational and diagrammatic conventions: one can (and indeed should) identify action-reaction pairs by subscripts (i.e. F_{12} and F_{21}) and by somehow identifying the pairs on the diagram, for example by circling the two forces and connecting them by a dotted line between the two free-body diagrams.

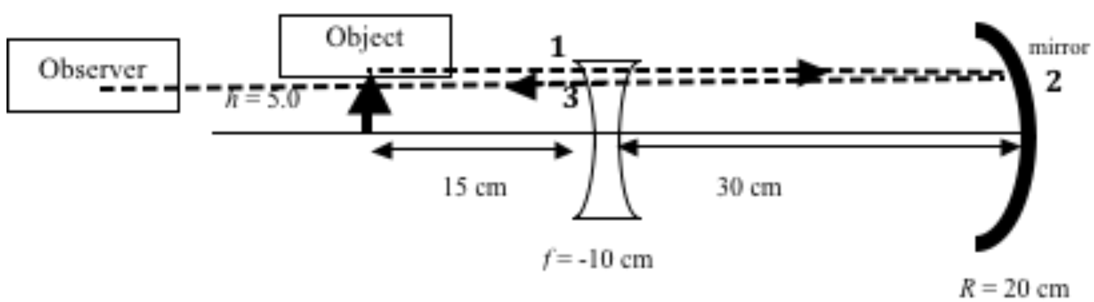
The following example illustrates the organizational power of a simple gesture.

Example 2. *gesture*

> *An object is placed 15 cm to the left of the optical apparatus shown below. Find the image location, type, attitude, magnification and image height for the combination below.*



Many students process the rays through the lens, reflect from the mirror and consider the problem finished, forgetting to send the ray back through the lens to the observer. A simple elaboration of this problem representation in the form of a gesture—a line tracing the path of a ray from the *Object* to the *Observer*—delineates the plot structure of the solution:

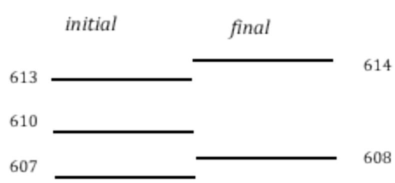


It is now obvious that the ray must be processed 3 times (**1.** lens, **2.** mirror, **3.** lens) before reaching the observer: the plot is tripartite.

Example 3. *a spatial representation*

> *You hear three beats per second when two pitches are generated. The frequency of one pitch is known to be 610 Hz. The frequency of the other pitch, pitch 2, is increased until you hear 2 beats per second. What is the new frequency of pitch 2?*
>
> *A. 608 Hz.*
> *B. 612 Hz.*
> *C. Either A or B.*

The following diagram helps students to organize their knowledge by translating a difference in frequency into more easily accessible spatial differences—it is a visual metaphor, and quite a useful one.



From this point the solution easily follows: the frequencies of the two lines in the *final* state have been increased, as required by the question in the problem. The beat frequency is the difference between one of these and the 610-Hz sound—only the lower pitch will generate 2 beats/s:

> Generally speaking, successful problem solving involves the ability to decipher correspondences between sets of signs (and what they stand for), and representing them appropriately in order to grasp solution strategy. (Danesi 2008, 10))

Example 4. *ray diagrams*

Students are required to draw ray diagrams on assignments and evaluations—and receive credit for their proper construction—but need to be reminded of the utility of this particular diagrammatic representation in problem solution. The following example, solved without a diagram, quite often produces classic novice errors:

> *The height of a real image formed by a curved mirror is four times the object's height for s = +30.0 cm. Find the radius of curvature for this mirror.*

The classic novice error in this problem is to write $M = h'/h = 4$ and proceed with a purely algebraic (and incorrect) solution. A quick sketch of a ray diagram alerts students to the fact that a "real image" is inverted, and thus M = -4, not +4. The absence of a diagram and the consequent error well illustrates Lemke's point about the crucial function of non-textual media:

> Visual figures in scientific text, and mathematical expressions also, are generally not redundant with the verbal Main Text information. They do not simply "illustrate" the verbal text, they add important or necessary information, they complement the Main Text, and in many cases they complete it. (Lemke 1998, 105)

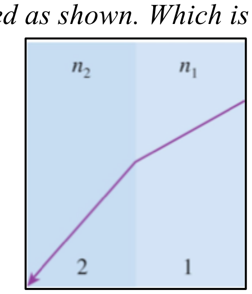
This example raises another interesting point: students (and teachers) are too often lax about the algebraic conventions of sign: a positive should not be assumed; we should require its inclusion because sign (itself a multimodal semiotic device) carries meaning. Thus if forced to write M=+4 a student may think to verify that the image is indeed upright.

concept questions: the ultimate multimodal arena

The best students can answer challenging concept questions with their eyes closed—the best students barely require teaching. Many students attempt to answer conceptual questions by sheer mentation, and often fail. Solving concept questions often involves the determination of conceptual relations, and a well-wrought representation exposes the essential features of a problem. Sketches of graphs, free-body diagrams, ratios and physical gestures (think of the right-hand rule) are the natural tools to solve all but the simplest of conceptual questions, as are manipulations of the appropriate formulae and diagrammatic annotations.

Example 5. *gesture*

A laser beam passing from medium 1 to medium 2 is refracted as shown. Which is true?



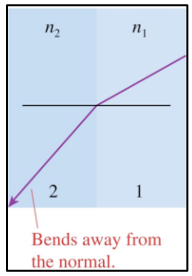
A. n1 < n2.

B. n1 > n2.

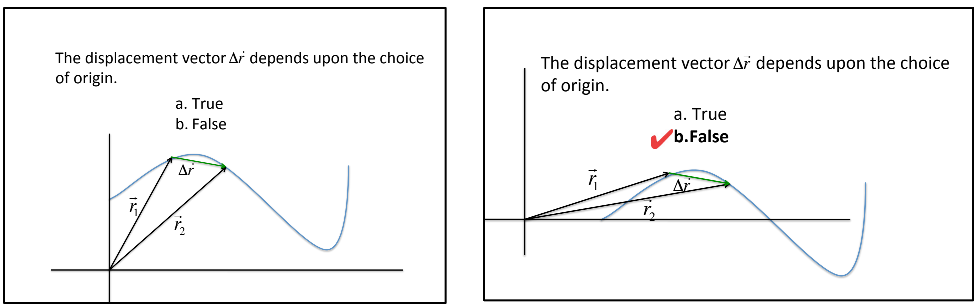
C. There is not enough information to compare

(Knight 2012, QuickCheck 23.4 Slide 23-53)

For many students simply sketching a normal invokes a gestalt switch, and the answer is now obvious:

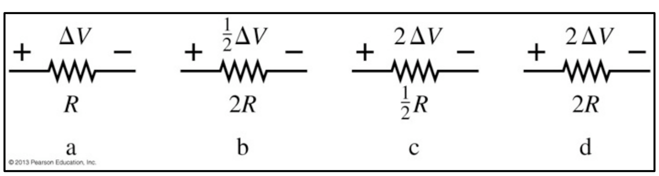


Example 6. *a change of axes*



Example 7. *ranking tasks*

> *Rank the four situations below according to the power dissipated in the resistors, from least to greatest.* (figure CQ31_06 Knight 2012, 914)

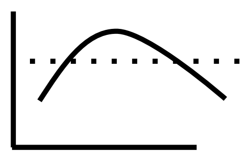


The use of ratios translates this problem into a more manageable form: apply the formula $P = \dfrac{\Delta V^2}{R}$ to each of the four options and compare the numerical factor multiplying $\dfrac{\Delta V^2}{R}$, thus reducing the question to the following: "Rank the numbers 1 , 1/8 , 8 and 2."

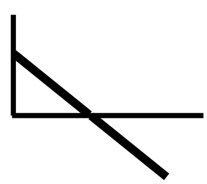
It is not always immediately apparent to students which modality to use: the murder board approach encourages them to solve the problem using several modalities.

Example 8. *If the average velocity of a particle over some time interval is zero can there be a nonzero acceleration during this interval? Justify your answer.*[15]

To justify their response students can sketch a parabola on an *x versus t* graph and show a horizontal secant (v_{av} = slope of this secant = 0 m/s):



or they could sketch a line on a v-t curve symmetric about the point v = 0 m/s and explain that for such motion v_{av} = 0 m/s:



In both cases the students need label their graphs and state why $a \neq 0$ m/s^2.

Example 9. *A fish inside a rectangular aquarium sees a dog that is standing 1.0 m in front of the aquarium glass. To the fish, the distance to the dog appears to be*

A. less than 1.0 m
B. greater than 1.0 m
C. equal to 1.0 m

Here one may simply use the equation for refracting surfaces, or a sketch of image formation. While we may prefer the elegance and simplicity of a diagram showing the formation of a virtual image from the deflected rays, students generally prefer the equation and should be taught that using a second modality serves to verify the results of the equation.

[15] I have found that the word "explain" can lead to exclusive verbal text (one student actually asked if she could use an equation to explain), whereas the word "justify" seems to indicate to students that they can use any combination of text, graphical relation, equation, sketch, etc.

Example 10. *graphs are not just for labs*

Many concept questions rely upon graphical interpretation, requiring students to derive information using slopes, intercepts or the nature of the relations (linear, quadratic, exponential, etc). The graph could include several curves representing experimental data for the spring force (F_{spring} versus Δs), the photoelectric effect (K_{max} versus f), or free fall (v_y versus t), to name but three. We can cull information from the fact that lines have slopes of different magnitudes (different spring constants) or the same slope (the photoelectric effect and free fall) or from the vertical and horizontal intercepts. Students seem to expect to use graphical modalities in the laboratory (whether or not they comprehend the manipulations they are instructed to perform during the lab is another matter) but shy away from their use in problem solution, to their detriment, for often graphical modalities are required in problem solutions to both *construct* and *transmit* meaning. Which last term brings me to the next topic.

Transmitting Meaning: (Re)Construction of Meaning by the Reader

the tricks of the trade: annotation, punctuation and page layout

Students *construct* meaning for themselves at the murder board—using the multimodal tools of the trade—during problem solution or when trying to understand a concept or method, be it in the textbook, the lecture hall or while writing their notes. They then *transmit* that meaning in their solution. But meaning is not just created and then delivered: the point of much semiotic analysis is to alert us to the role of the reader, and it is the reader who must construct meaning (and for student solutions the reader is the marker). A reader of scientific literature—and this is a key point—constructs meaning from all semiotics—both print and images—relying on the former as much as the latter.

Important information is communicated to the reader at levels other than the usual graphics and text. Lemke's analysis of scientific texts reveals three generalized semiotic functions for constructing meaning, namely *presentational, orientational and organizational*. The meaning of scientific communication is carried not just by equations and charts, tables, graphs etc. but also by the choice of relative font size in titles, headings and labels, as well as the paragraph and section structure: ". . . geometric relations of figure space to caption space tell us which elements are to be preferentially read in relation to which other elements; what goes with what." (Ibid. 95)

One might assume the affect of such semiotics to be tertiary, at best. But in fact research reveals their import for cognitive processing:

> The effects of labeled illustrations on guiding attention is indicated by the illustrations group recalling more explanative than non-explanative information relative to the control groups in both Experiments 1 and 2. The effects of labeled illustrations on building connections is indicated by the illustration group outperforming the control groups on problem-solving transfer . . . Providing only pictures (without corresponding labels) or only labels (without corresponding pictures) did not allow students to build useful mental models of the system as indicated by problem solving transfer, whereas students given labeled illustrations performed much better . . . without a coherent diagram that integrated the information, students performed relatively poorly on problem-solving transfer. These results help to clarify further Larkin and Simon's (1987, p. 65) analysis of "why a diagram is (sometimes) worth a thousand words. (Mayer 1989, 244)

The meaning carried by labels etc. pertain not just to textbooks and solutions but also to the very problems that students are required to solve:

> . . . many advocate teaching learners metacognitive strategies designed to activate one's schema before reading, such as reading heading and the title, looking a visuals in the text, and making predictions based on the title and pictures. . . (Widmayer 2005, 2)

Students need to use such semiotics in their problem solutions. Imagine trying to read from an article or textbook with no formatting whatsoever: no bold or italics, no attention to the position and relative size of diagrams/text/figures/equations, no labeling or numbering of these, nothing: or, a novel without punctuation (well, it's been done). Sadly this, plus an element of spatial disarray, is what many students submit as a solution. While we cannot expect from our students the sophistication of text setting used in textbooks and journal articles, we *can* teach them to use some of the more obvious semiotic tricks of the trade that allow readers (markers) to properly construe and construct their intended meaning. To this end I define the following three semiotic categories: *annotation*, *punctuation* and *page layout*.

- **annotation**
  - The most important annotation is a brief statement of the theme at the outset of the solution.
  - Textbooks number equations and use those numbers as referents, and this students can easily do, post-murder board analysis. Note that given the time constraints of an exam, students must often punctuate on the fly, numbering a previously derived equation when they realize they would like to refer to it.

  Other examples of annotation:

    - brief descriptive labels for various sketches or graphical representations
    - an appropriate coordinate system, labeled
    - an indication of the direction of the acceleration or net force
    - arrows indicating quantities that take on certain values (usually 0, 1 or ∞)
    - appropriate connective statements between, for example, figure and graph, or (most often) equations and between interim results

      "and thus", "from equation # . . ." , "it follows that . . .." etc
- **punctuation**
  - underline interim and box final results
  - bracketing or hierarchical indentation to indicate structure
  - bracketing when making a substitution in equations, *e.g.*

$$\begin{aligned} R &= rN \\ &= r(N_0 e^{-rt}) \end{aligned}$$

- **page layout**

For many problem types I find it best to split the page into two or three vertical columns. This may seem an obvious ploy, but not so to many students (else we would not encounter so many poorly-rendered solutions). Such a page layout works well for rendering the following:

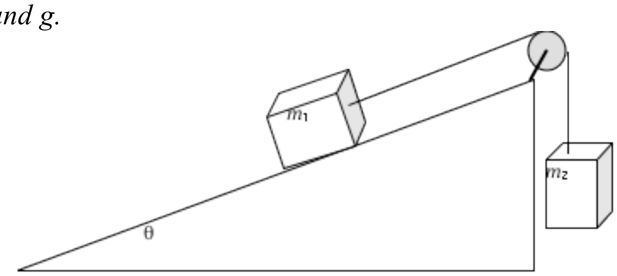
- bipartite plot structures, regardless of the underlying theme or themes

- one column for the first part, another for the second, with a clear indication of the transition between parts (*e.g.* final state of part 1 becomes the initial state of part 2)

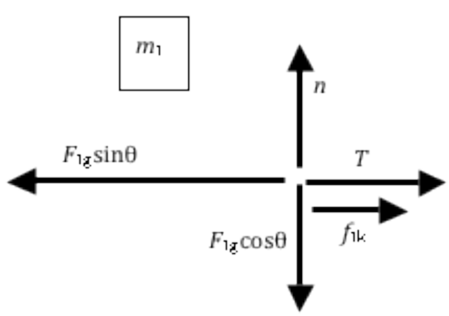
- conservation themes (initial and final states)
- Dynamics, Momentum and other problems involving vector quantities
  - the separate columns for the components serve to simplify the parsing of the meaning, and this for both the student and the teacher
- Dynamics problems with coupled motion of two or more objects (for example, block/pulley/string questions)

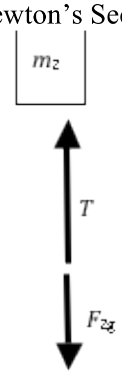
examples of annotation, punctuation and page layout

Example 1. *Two blocks are connected over a light, frictionless pulley as shown below. The coefficients of friction between the inclined plane and the mass are* μ_k *and* μ_s*. Assume that the block on the inclined surface slides down that surface. Find an expression for the magnitude of the acceleration of the system in terms of the given variables and g.*



The problem-solving process for this question begins with free-body diagrams (a translation from story level to plot/thematic level) followed by statements of Newton's Second Law for each mass:





$\sum F_x = ma_x$ $\quad$ $\sum F_y = ma_y$ $\quad$ | $\quad$ $\sum F_y = ma_y$

- **page layout**

The page is subdivided to separate the free-body diagrams and the attendant statements of Newton's Laws for each object. Given the space restrictions of a portrait 8.5"x11" page students often subdivide into two rows rather than two columns.

- **punctuation**

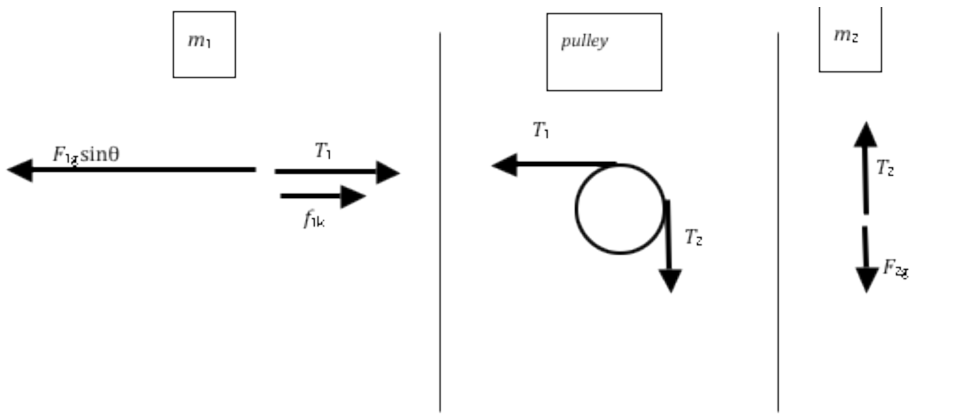
Underlining the statements of Newton's Second Law as headings serves to organize the subsequent algebra: beneath these title headings one writes the algebraic equations that can essentially be read off of the corresponding free-body diagrams.

- **annotation**

(Note that for clarity one should add the direction of the acceleration and the coordinate axes.)

Mass m_1, and only this mass, appears in the left-hand column; m_2, the right. The normal, gravitational and friction forces must be given corresponding subscripts, n_1, n_2, f_{k1} *and* f_{k2} etc. A common novice error in this type of problem is simply to write, for example, f_k in each column and then in subsequent algebraic manipulation treat all friction forces as the same quantities: both marker and student are uncertain as to which values are to be used in the determination of friction or the normal. (In a case where kinetic friction acts on both masses the normal forces used to determine friction are not the same, even if by coincidence they are numerically equal.)

Example 2. The previous example involved a "light" pulley. In the case of a massive pulley the following page layout could be used:



$\underline{\sum F_x = ma_x}$ $\underline{\sum F_y = ma_y}$ $\underline{\sum \tau = I\alpha}$ $\underline{\sum F_y = ma_y}$

- **page layout**

For dynamics problems involving interacting objects I subdivide the page into *n* columns, where *n* is of course the number of bodies. For problem solving purposes (see **annotation** below) I use the same left-to-right order for the free body diagrams as is found in the surface-level (story) diagram for the problem.

- **annotation**

The left to right order of the free-body diagrams allows for a quick visual verification that the tensions have been properly labeled and that these internal forces, with the action-reaction pairs pointing in opposite direction, will indeed cancel when one sums the equations.

- **punctuation**

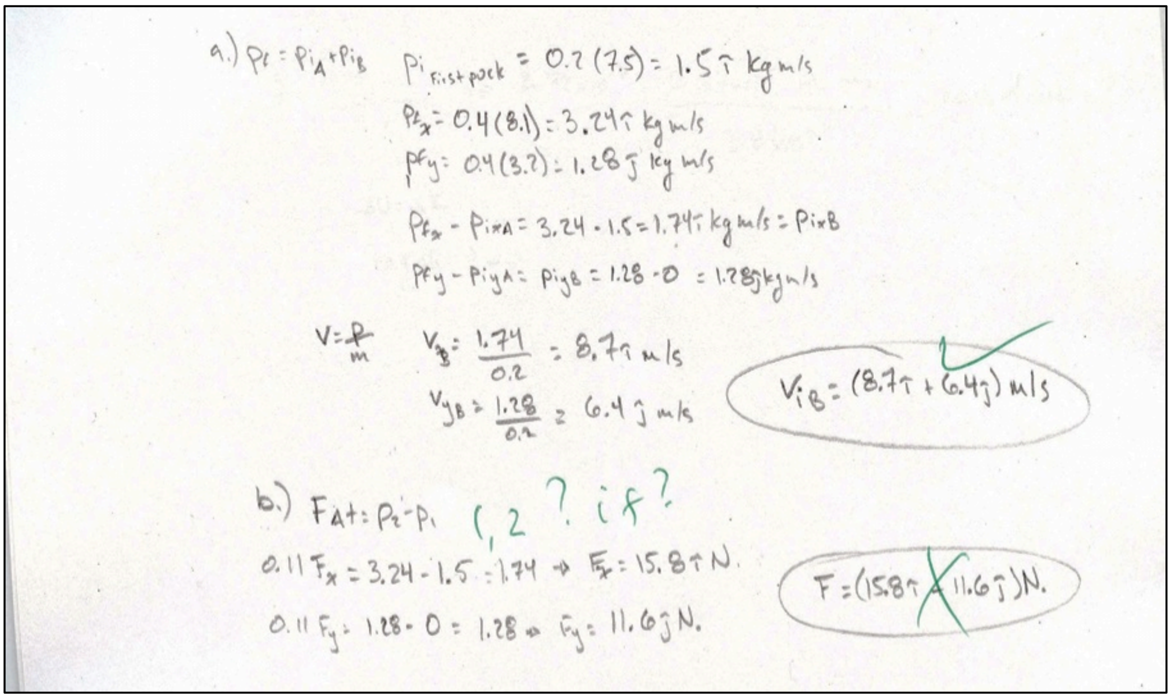
As before, each statement of Newton's Law functions as a heading under which the appropriate algebraic expression is formed.

Example 3. *A 200-g puck, moving along a frictionless, horizontal table, has a velocity of* $7.5\hat{i}\,m/s$ *when it is struck by a second, identical puck. The two pucks stick together and move off with a velocity of* $\left(8\hat{i}+3.2\hat{j}\right)m/s$. *The duration of the collision is 0.10s.*

a. Find the velocity of the second puck before the collision.

b. What is the average force exerted on the first puck during the collision?

A student solution is shown below.



a.) $p_f = p_{iA} + p_{iB}$ $\quad$ $p_{i\,\text{first puck}} = 0.2(7.5) = 1.5\,\hat{i}$ kg m/s

$p_{fx} = 0.4(8.1) = 3.24\,\hat{i}$ kg m/s

$p_{fy} = 0.4(3.2) = 1.28\,\hat{j}$ kg m/s

$p_{fx} - p_{ixA} = 3.24 - 1.5 = 1.74\,\hat{i}$ kg m/s $= p_{ixB}$

$p_{fy} - p_{iyA} = p_{iyB} = 1.28 - 0 = 1.28\,\hat{j}$ kg m/s

$V = \frac{p}{m}$ $\quad$ $V_{xB} = \frac{1.74}{0.2} = 8.7\,\hat{i}$ m/s

$V_{yB} = \frac{1.28}{0.2} = 6.4\,\hat{j}$ m/s

$V_{iB} = (8.7\,\hat{i} + 6.4\,\hat{j})$ m/s

b.) $F\Delta t = p_f - p_i$ $\quad$ (1, 2 ? i f ?

$0.11\,F_x = 3.24 - 1.5 = 1.74 \rightarrow F_x = 15.8\,\hat{i}$ N.

$0.11\,F_y = 1.28 - 0 = 1.28 \rightarrow F_y = 11.6\,\hat{j}$ N.

$F = (15.8\,\hat{i} + 11.6\,\hat{j})$ N.

- **page layout and punctuation**

While I would prefer a layout of two columns for the *x*- and *y*-directions there is certainly no problem comprehending the organization of the solution for part a.: the punctuation and hierarchical indentation allow us to easily follow the proof.

- **annotation**

This is where the student runs into trouble, in part b. Whether the student falls victim to sloppy labeling, or if the sloppy labeling is symptomatic of a conceptual weakness we can not say: are they using $\vec{F}_{av} = \frac{\Delta\vec{p}_1}{\Delta t}$ or $\vec{F}_{av} = \frac{\Delta\vec{p}_2}{\Delta t}$? We don't know and, apparently, neither do they.

Example 4. An object is launched with some initial speed. Find the maximum height of the object.

There are of course innumerable variations on this problem that could include friction, the angle of an incline, a spring, etc.

- **page layout and punctuation**

For any of these variations the solution begins with a statement of the theme:

$$\underline{W_{ext} = \Delta E}$$

For the case where there are no external forces acting on the system a statement such as:

In the absence of external forces, $\underline{\Delta E = 0}$

and then a restatement of the Conservation of Mechanical Energy, punctuated as a heading, to initiate the algebraic phase of the solution:

$$\underline{E_f = E_i}$$

- **annotation**

At this point novice students will want to write

$$mgy = \tfrac{1}{2}mv^2$$

in the now-familiar headlong drive toward a numerical answer. I would rather that they use the modality of equations, with proper annotations, to communicate physical information thusly:

$$(\cancelto{0}{K} + U)_f = (K + \cancelto{0}{U})_i$$

where $K_f = 0$ J since the particle comes to rest at the maximum height

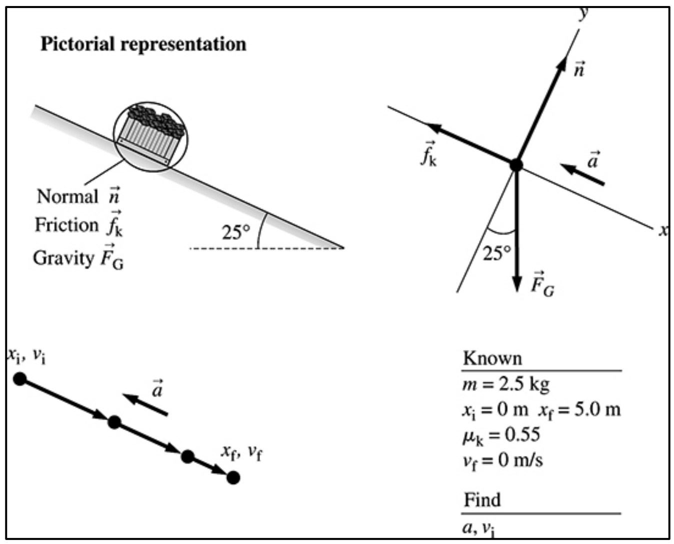
$U_i = 0$ J, taking the lowest point as the zero of potential energy

Labeling of the initial and final states, the downward gesture to denote a quantity that is zero and the various annotations and punctuations all serve to transmit meaning to the marker, making it evident that the student understands this particular realization of the principle of the Conservation of Energy. Crucially for students this careful work allows them to organize their knowledge in a systematic way that may well cue them to the solution of more complex and demanding problems.

At this point one can proceed to insert (properly annotated) expressions for K and U:

$$mgy_f = \tfrac{1}{2}mv_i^2 \,.$$

Example 5. The following textbook solution shows a typical sequence of reductive diagrams involved in the problem-solving process (P6 .44 Knight, 2008, 6-29.)



Here we are asked to determine the initial speed required to carry a box of nails just to the edge of a roof, where it stops.

- **page layout**

The first diagram (upper left-hand corner), at the story level, is a literal representation of the situation; the second (upper right-hand corner), a free-body diagram showing the direction of the acceleration and the required axes, is at the thematic level (Dynamics); the third diagram is also at the thematic level (lower left-hand corner, Kinematics With Constant Acceleration). The bipartite plot could have been made more transparent (revealed to the reader) by using a template showing second and third diagrams side-by-side, labeled in some fashion (such as **1. Dynamics** and **2. Kinematics)** that would allow the reader to almost instantaneously construct meaning: this is a bipartite problem, I know what to expect beneath each part, and so the entire solution (though not the details) can be taken in as a whole.

The Content Of The Form: Solution Template As A Global Semiotic

The above "tricks of the trade" serve to expose the narrative. For the purpose of plot delineation I find page layout to be a crucial semiotic device: the global organization of content affects the manner in which the reader parses it. Of course this parsing (and meaning making) need not be linear: scientific texts, as opposed to spoken language, are not meant to be read according to some implied sequence: visual semiotics ". . . are at least two-dimensional and any one-dimensional sequence represented in two dimensions can be accessed at any point at any time." (Ibid. 95) Scientists reading a report may jump from abstract to references, skim the tables and figures first, look at graphs, captions etc, and only then proceed to the main text: such " . . . are the habits of expert readers, those who could themselves have written this text or one very like it." (Ibid. 96)

Instructors are just such expert readers, and students need to keep their audience in mind: they need to be taught to format their solutions to allow for easy parsing by their instructor, solutions that communicate their conceptual grasp (or lack thereof) of the material at hand. Good student solutions can often be read as a gestalt[16]—the details of the solution can be taken in at a glance. How is this accomplished? I can often skim a solution and know that the student fully comprehends merely by the physical layout of the solution—the format of the page, the narrative that is rather explicit in the spatial hierarchy[17]. The spatial hierarchy of *page layout* is further adumbrated by *punctuation* and *annotation*—this is what I mean when I refer to "The Content Of The Form."

Taken together, the three semiotic devices of *annotation, punctuation* and *page layout*

[16] *Gestalt* is German for *form* or *shape*, the idea being that with a glance we can comprehend that the student has correctly represented the problem—to mark the solution we need only check the algebraic details.
[17] The various levels of a hierarchy imply a logical sequence—a plot. For hierarchy as a species of narrative see Branigan 1992, 16.

constitute what I call the *solution template.* There are several advantages to teaching and modeling the use of templates:

- meaning making for the reader/marker
  - gestalt

    The entire solution can be taken in as a whole: we know at once that the solution is of the correct form (or not) and can proceed to check the algebra.
  - diagnostic

    The layout allows for quick recognition of student problems (is there a problem with the physical theory, the algebra or a numerical calculation).
  - marking

    If the student has chosen and states the incorrect theme then of course the marking process will be rather brief. Teachers generally penalize less for algebraic or numerical mistakes more for errors in physical reasoning. It is very easy to grade a well-wrought solution, correct or no.
- meaning making (conceptual clarity) for students
  - Templates explicitly privilege communication and help foster better intellectual organization and a meticulous attention to detail that many students sorely lack.
  - Templates encourage the forward-working habit of experts: a template really must begin with a statement of the theme (as experts do in their think-aloud protocols).
  - Modeling templates in the lecture reinforces the physical concepts (the themes) and problem-solving techniques that we are most interested in teaching.
  - The main objective of a solution, as with written text, is communication, *not* merely finding the answer: solution templates are antithetical to the chug-and-plug heuristic.
  - While there can be no objection to a full statement of a problem solving strategy (à la Leonard et al, 1996) the various semiotics in the solution template have here rendered such a lengthy written text all but unnecessary.

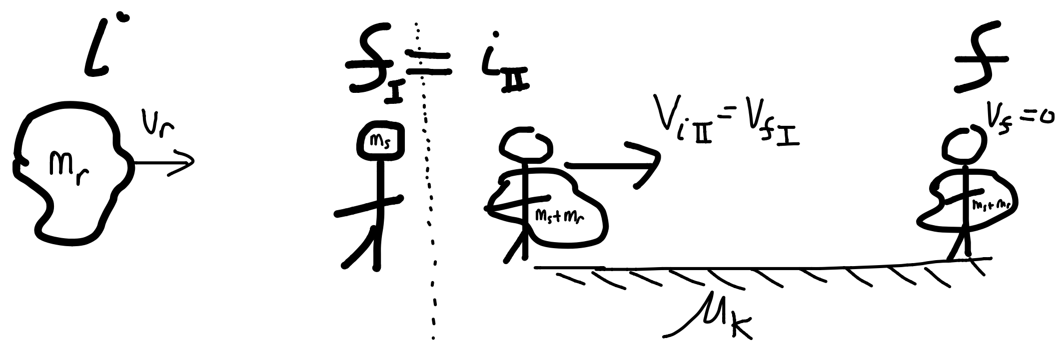
examples (continued)

Below a single template is used to solve any number of bipartite problems where the final state of the first part of the question becomes the initial state of the second.

Example 6. *A 30-kg sack of rutabagas strikes a stationary, 60-kg ice skater. After impact, the skater and rutabagas travel 10.0 m on the ice before coming to rest. If $\mu_k = 0.50$, find the initial horizontal speed of the offending tubers.*

- **page layout**

The page layout must show some explicit statement about the relationship between the final state of part one and the initial state of part two. There are a number of ways to accomplish this, but what I like about the following template is the vertical alignment between the various modalities (annotated sketch, parts I and II, the equations).



Part I: Conservation Of Momentum | Part II: Conservation of Energy

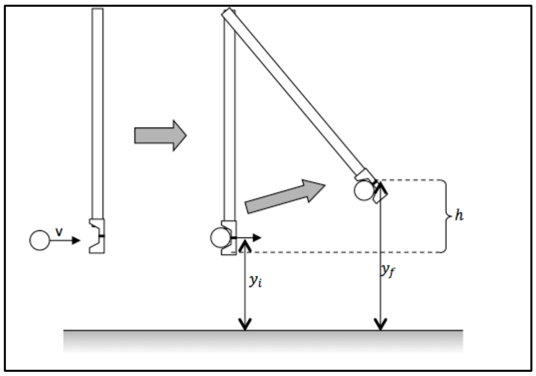
$\underline{\vec{\mathbf{p}}_f = \vec{\mathbf{p}}_i}$ | *final*$_{\text{part I}}$ = *initial*$_{\text{part II}}$ | $\underline{E_f = E_i}$

- **punctuation and annotation**

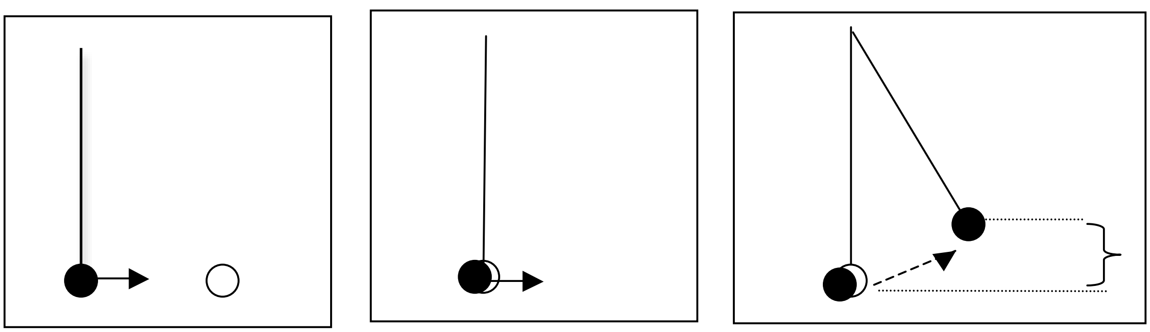
Underlining is *de rigueur*, but the most important semiotic functions are carried here by the annotations of the masses and the initial and final states. (Anecdote: As I edit this document a student studying for the final exam came to ask me about this very question. “I’m trying to find

v_i. and am not sure what equation to use” Of course I responded that rather than searching for an equation he should be solving the problem, beginning with theme.)

Example 7. The same solution template works well for the *ballistic pendulum*:



One of many possible realizations of the page layout involves a reimagining of the sequence into three separate diagrams.

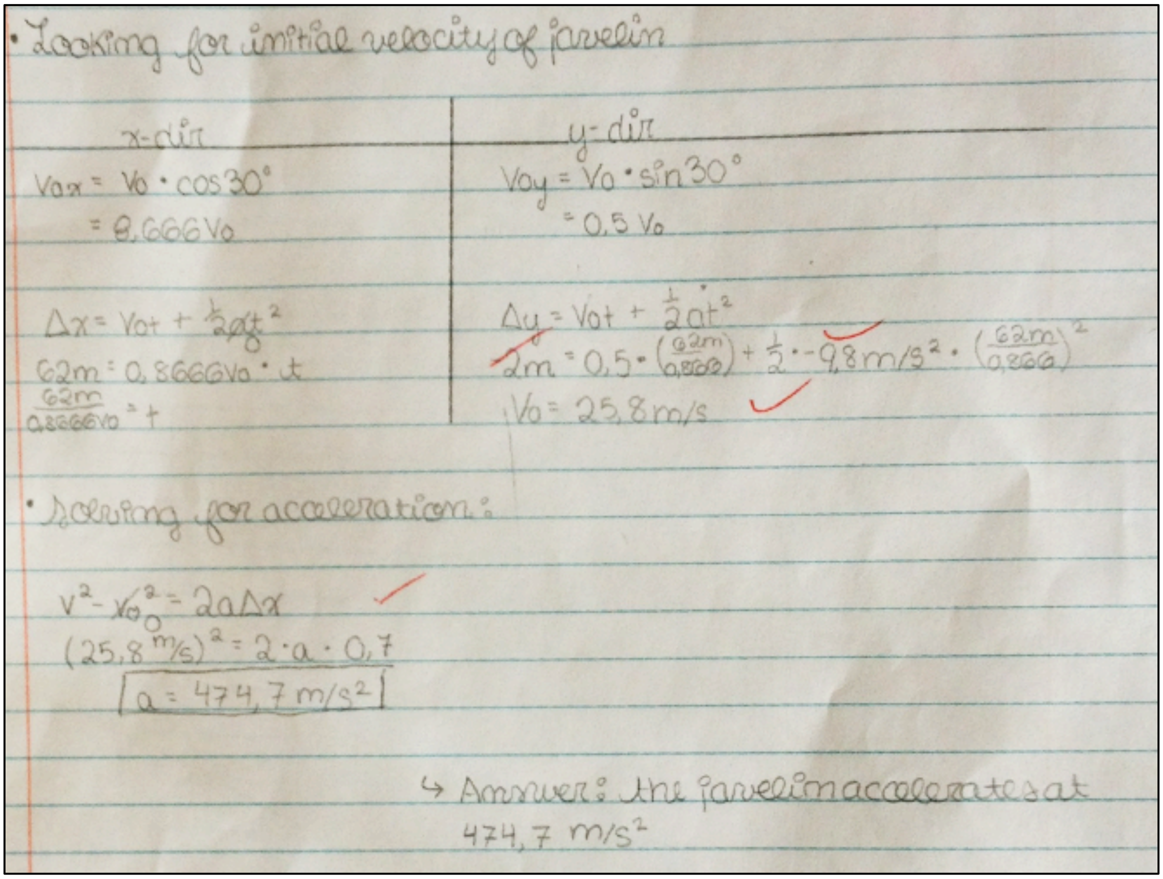


Example 8. For more straightforward questions many students can and will just plunge in and begin solving. The following question is assigned before I have talked to students about solution templates:

> *A javelin thrower standing at rest holds the center of the javelin behind her head, then accelerates it through a distance of 70 cm as she throws. She releases the javelin 2.0 m above the ground traveling at an angle of 30 degrees above the horizontal. . . In this throw, the javelin hits the ground 62m away. What was the acceleration of the javelin during* the throw? Assume that it has a constant acceleration. (Knight 2012, 113)

The two student solutions given below betray a nascent grasp of solution templates.

student solution 1



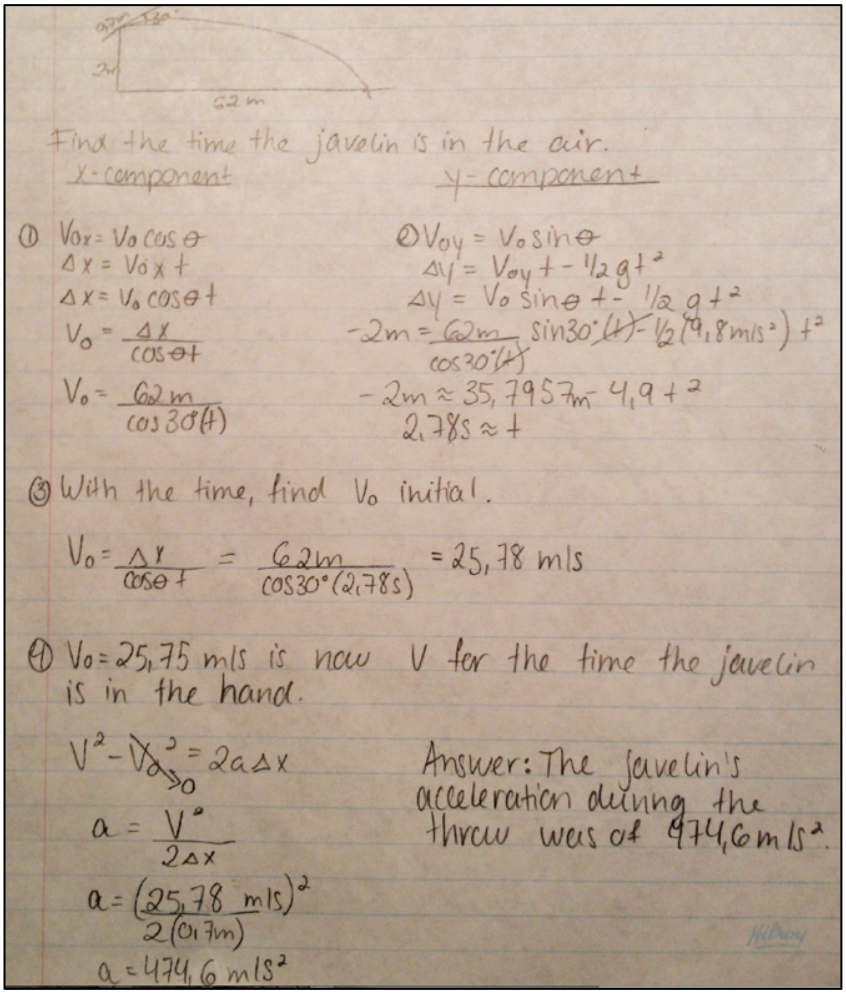
- **page layout**

The bipartite nature of the solution is communicated by the student's actional phrases "Looking for initial velocity of javelin" and "Solving for the acceleration." There is no acknowledgment of the two different accelerations used in the two parts, but this is a satisfactory solution presented in a polished but nonetheless novice style where the idea of the solution is to find the answer rather than to communicate knowledge of physics.

- **punctuation and annotation**

The minimal underlining and subscripts serve the student well in what is a fairly straightforward question.

Student solution 2



0.7 m, 30°, 2 m, 62 m

Find the time the javelin is in the air.

x-component | y-component

① $V_{0x} = V_0 \cos\theta$

$\Delta x = V_{0x} t$

$\Delta x = V_0 \cos\theta\, t$

$V_0 = \frac{\Delta x}{\cos\theta\, t}$

$V_0 = \frac{62\,m}{\cos 30°(t)}$

② $V_{0y} = V_0 \sin\theta$

$\Delta y = V_{0y} t - \frac{1}{2} g t^2$

$\Delta y = V_0 \sin\theta\, t - \frac{1}{2} g t^2$

$-2m = \frac{62m}{\cos 30° \cancel{(t)}} \sin 30° \cancel{(t)} - \frac{1}{2}(9{,}8\,m/s^2)\, t^2$

$-2m \approx 35{,}7957m - 4{,}9 t^2$

$2{,}78 s \approx t$

③ With the time, find V_0 initial.

$V_0 = \frac{\Delta x}{\cos\theta\, t} = \frac{62m}{\cos 30°(2{,}78s)} = 25{,}78\ m/s$

④ $V_0 = 25{,}75\ m/s$ is now V for the time the javelin is in the hand.

$V^2 - \cancel{V_0}^2 = 2a\Delta x$ ($V_0 \to 0$)

$a = \frac{V^2}{2\Delta x}$

$a = \frac{(25{,}78\ m/s)^2}{2(0{,}7m)}$

$a = 474{,}6\ m/s^2$

Answer: The javelin's acceleration during the throw was of $474{,}6\ m/s^2$.

- **page layout**

This student also uses two columns, for the *x* and *y* components, while dividing the page not according to physical theme but rather according to mathematical calculations. Again, for a relatively straighforward question such as this the strategy is successful: I understand what they are doing and need not invoke my knowledge of physics to fill in important gaps.

Example 9. *A textbook solution* (supplied with Knight, 2012)

The following is not included in the textbook but rather with the instructor resources. At any rate, it *is* meant for student consumption.

4.51. Model: Assuming constant acceleration allows us to use the kinematic equations.
Visualize: We apply the kinematic equations during the free-fall flight to find the velocity as the javelin left the hand. Then use $v_f^2 = v_i^2 + 2a_s\Delta s$ where $\Delta s = 0.70\text{m}$.

Solve: The range is $\Delta x = 62\text{m}$.

$$\Delta x = (v_0)_x \Delta t = v_0 \cos\theta \Delta t \Rightarrow \Delta t = \frac{\Delta x}{v_0 \cos\theta}$$

$$y_f = y_i + (v_0 \sin\theta)\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

Insert our new expression for Δt.

$$\Delta y = (v_0 \sin\theta)\frac{\Delta x}{v_0 \cos\theta} + \frac{1}{2}(-g)\left(\frac{\Delta x}{v_0 \cos\theta}\right)^2$$

$$\Delta y = (\tan\theta)\Delta x + \frac{1}{2}(-g)\left(\frac{\Delta x}{v_0 \cos\theta}\right)^2$$

$$\frac{1}{2}(g)\left(\frac{\Delta x}{v_0 \cos\theta}\right)^2 = (\tan\theta)\Delta x - \Delta y$$

Solve for v_0.

$$v_0^2 = \frac{g}{2}\left(\frac{\Delta x}{\cos\theta}\right)^2\left(\frac{1}{(\tan\theta)\Delta x - \Delta y}\right)$$

$$v_0 = \sqrt{\frac{g}{2}\left(\frac{\Delta x}{\cos\theta}\right)^2\left(\frac{1}{(\tan\theta)\Delta x - \Delta y}\right)}$$

$$= \sqrt{\frac{9.8\ \text{m/s}^2}{2}\left(\frac{62\ \text{m}}{\cos 30°}\right)^2\left(\frac{1}{(\tan 30°)(62\ \text{m}) - (-2\ \text{m})}\right)} = 25.78\ \text{m/s}$$

This is the speed as the javelin leaves the hand. It now becomes v_f as we consider the time during the throw (as the hand accelerates it from rest).

$$a = \frac{v_f^2 - v_i^2}{2\Delta s} = \frac{(25.78\ \text{m/s})^2 - (0\ \text{m/s})^2}{2(0.70\ \text{m})} = 470\ \text{m/s}^2$$

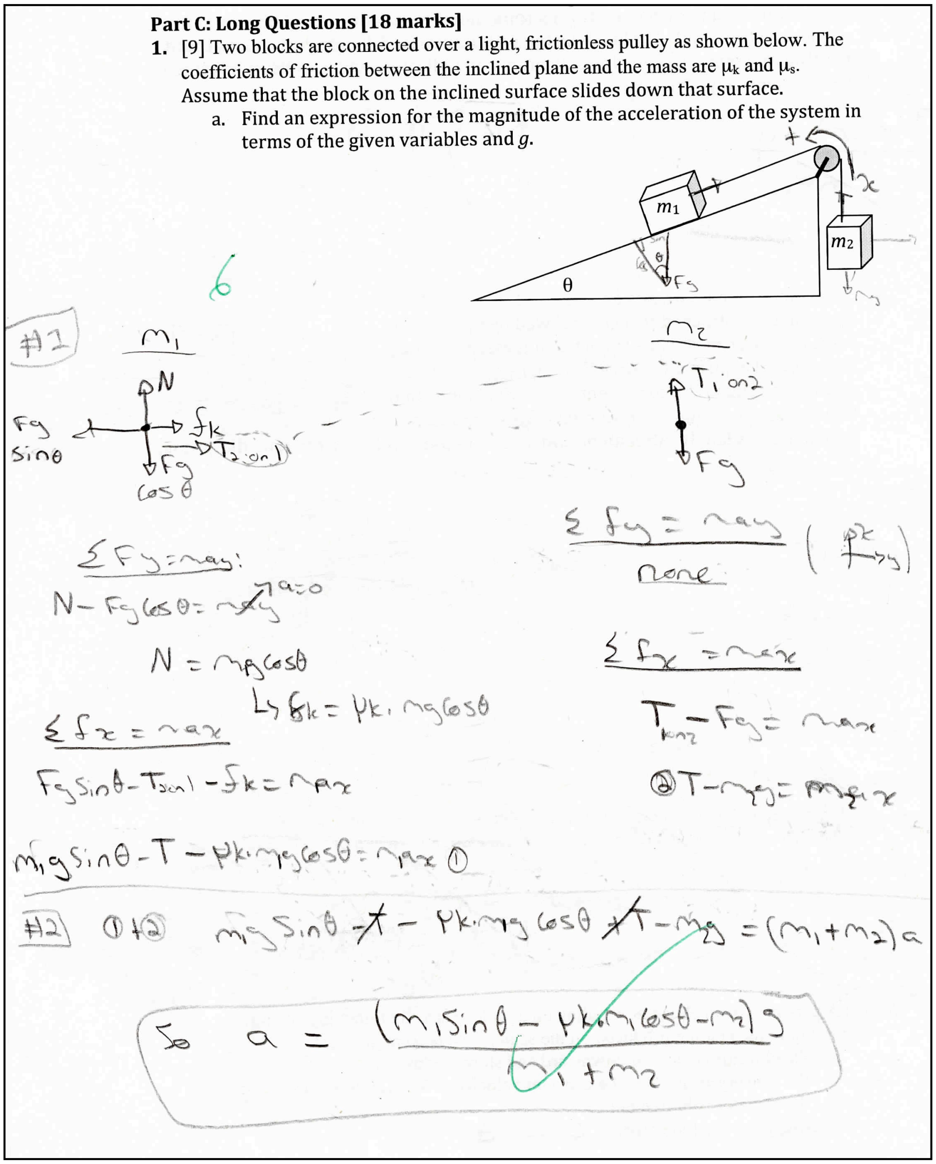
Assess: This is a healthy acceleration, but what is required for a good throw.

This solution features no diagram and little other than an implied template. One *can* ferret out the crucial statement of plot structure:

> "This is the speed as the javelin leaves the hand. It now becomes v_f as we consider the time during the throw . . ."

But one should not have to rummage around a solution (particularly from a pedagogical source) to construct meaning. This failing is no doubt in part due to the economics of textbook publishing.

Example 10. Consider another student solution to the second question presented in the introduction.


Part C: Long Questions [18 marks]

1. [9] Two blocks are connected over a light, frictionless pulley as shown below. The coefficients of friction between the inclined plane and the mass are μ_k and μ_s. Assume that the block on the inclined surface slides down that surface.

a. Find an expression for the magnitude of the acceleration of the system in terms of the given variables and g.

- **page layout**
  - There is no statement of the theme, but the page layout is exemplary.
  - Numbering (**#1** and **#2**) splits the page into horizontal rows that correspond to different stages of the problem solution (#1 contains all of the physics, #2 the algebraic manipulation). I stress this distinction in class: clearly most of the marks are assigned to #1. Otherwise the student follows the solution template in our Example 1 above.

- **punctuation and annotation**
  - progressive reduction of diagrams: some annotations on the story diagram, then free body diagrams with components resolved
  - the positive direction indicated at the pulley in the story diagram
  - correct labeling using subscripts 1 and 2 in the equations, the action reaction pair T (at this point in the course they are not aware that the tensions are in fact not action/reaction pairs), unfortunately not for the friction acting on mass 1
  - underlining of m1 and m2 to identity free body diagrams and the bipartite split of the page; underlining of the component statements of Newton's Second Law
  - What is missing from the annotations are appropriate connective statements: "and thus", "from equation # . . ." , "it follows that . . .." etc.

I have modeled and taught the three components of solution template (annotation, page layout and punctuation), and it is gratifying to see students routinely use these resources to solve problems and, better yet, to innovate and find a solution to a difficult problem.

The Need to Teach Multiliteracies

translating between modalities

Scientists know how to negotiate between the various modalities used in the scientific literature, students need to learn how to do this, and we assume that they do so through years of exercises using these modalities. But they are given no explicit instruction, learning from models in textbooks and lectures—a species of immersion, really—and such tacit learning is insufficient:

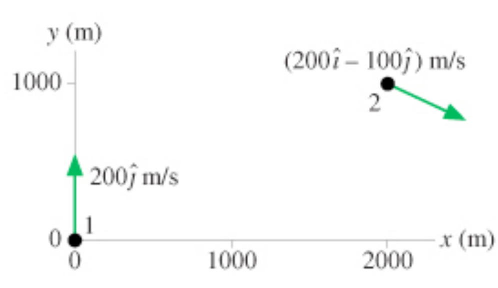
> Multiple representations are the tools that scientists use to construct new knowledge, solve problems, evaluate their work, and communicate. If we want our students to reason like scientists, we need to engage them in similar activities and convince them of the usefulness of representations. (Etkina & Van Heuvelen, 2008, 25)

At this point one could object that we already teach multimodal resources and do demand that students use these (the most obvious example being the free-body diagram). The question is, Do students really comprehend the power and utility of the various modalities? Consider the deceptively simple notation used in chemical reaction, for example the symbol $3H_2O$. The numerals 2 and 3 deftly encompass two meanings, two levels of interpretation: the 3 quantifies at the macroscopic level while the 2 carries detailed information about molecular structure. When reading or balancing chemical equations, however, students can lose sight of these meanings, conflating the two numerals while interpreting the reactive processes implied by the semiotic "+" in chemical equations as an algebraic operator:

> . . . students might mistake the chemical symbols of H and O for mathematical variables such as X and Y and thus conceive the nature of chemical reactions as mere mathematical calculations. (Liu 2009, 136)

Such conflation and reinterpretation can lead to serious conceptual difficulties in more advanced problems, so there are reasons for concern, even if students do properly balance chemical equations (see Liu 2009).

Translation between various modalities, and knowing which modalities to use, are key problem-solving skills. I assign the following exercise every year and find that it faithfully exposes student weakness with multimodal representations. The problem (which in the textbook is labeled as a mere exercise) that defeats so many is simply to find the acceleration vector given the initial and final velocity vectors (Knight 2012, 120):



Many students have difficulty comprehending just what information is being presented and are unable to translate from this representation to the more fertile one that uses equations: in other words, to interpret correctly the various modalities in the figure and translate the problem into a straightforward algebraic exercise. Perhaps they are expecting a trajectory on the *y* versus *x* graph and are confused by its absence? When students come to my office to ask about this question (and they invariably do) I try to cue various solution templates by sketching a trajectory on the graph (any will do, as long as the velocity vectors at points **1** and **2** are tangent to this trajectory). If this simple gesture is insufficient I will ask them to write the initial and final velocity vectors and then, for weaker students, the components of these vectors. At this point all of the students can proceed to the answer.[18]

constructing meaning

Clearly for many students both comprehension of physical theory and the ability to solve problems is compromised by a lack of facility with the various modalities:

> . . . current research on multimodality in science education pays scant attention to how to help students develop the diversified illiteracies . . . there exists an unwarranted assumption that students already have a good command of multiliteracies. But the opposite seems to be true . . . even college students and secondary school science teachers have a limited understanding of chemical diagrams and symbolism, which necessitates explicit instructions on multiliteracies in science education. (Liu 2009, 129)

Rare, it seems to me, is the student who fully utilizes the various modalities in problem solution, or is even aware of the conceptual richness and communicative power of these highly specialized symbols. They need to be taught how to use the meaning-making resources not merely to fulfill the professor's demand to draw a diagram but as a means for communicating concepts. Consider again the case of chemical equations:

> It therefore follows that in order to improve multiliteracies in chemistry, symbolism (as well as other types of representations) would better be viewed as meaning making resources rather than conventionalized codes, and teachers need to develop explicit instructions on the meaning making patterns of chemical representations. (Liu 2009, 138)

[18] A completely unnecessary confusion exists for some students because of the choice of numbers used in the problem: every year many students ask why it is that at x = 2000 m the x component reads 200 rather than 2000! These novice students are being miscued by surface features of the problem—there are more important issues to deal with here (multimodal translations) and this numerical distraction is naught than poor pedagogy, akin to using $2^2=4$ as a first example when introducing exponents (the reasonable prediction of many young students is that $2^3=6$)

When modeling solutions we need explicitly alert students this “meaning-making” function: it need come out from the tacit shadows and be explicitly integrated as part of the curriculum:

> When instructors use diagrams and other representations in the classroom, they should make their reasoning explicit so students can understand the underlying meaning associated with the specific features of these expressions and see how they function to support the solution of problems in chemistry.

schemata and getting unstuck

The third study of expert-novice categorization (Chi et al. 1981—discussed in Section 3 above) set out to determine the content of the schema invoked by categories. In terms of narrative elements the *schema* (taken as synonymous with knowledge structure) is the *theme* plus possible *solution templates*. The structure of the solution templates is itself contingent upon plot and motif, and thus the complex knowledge structure that is the schema involves a richly organized (one hopes) web of theme, plot, motif and solution template. I repeat here my concluding remark to that third study:

> In sum, this study demonstrates that categories invoke for experts a knowledge structure (schema) that includes potential methods of solution, whereas a novice schema invokes a few structural features, many surface features and no methods of solution.

The expert deployment of the various semiotic resources (diagrams, graphs etc.) at the murder board reveals—or at any rate suggests—plausible schema for problem solution: “Experts integrate multiple representations of ideas, whereas novices often have only one representation, or are unable to relate different representations.” (Gerace and Beatty 2005, 3) Playing around at the murder board with various modalities within solution templates cues experts to alternate schema, should their initial approach fail:

> . . . effectively representing and re-representing problems is an important problem-solving skill. And this skill includes being aware of the nature of representations and the possible need to re-represent a problem, especially if one is stuck and cannot identify the nature of the sticking point. (Maloney 2011, 14)

But such crucial cues are only available to students who they have been alerted to their existence: as noted in one paper, novices “ . . .'flailed about' when stuck, whereas the experts proceeded more systematically. (Kohl and Finkelstein 2007, 135). So we have yet another critical function of multimodal facility and another reason to give students explicit instruction in the multiple modes of representation and their manipulation in problem solution, namely as a means to cueing alternatives.

transmitting meaning

In their written solutions students need to make better use of the full array of multimodal tools and not rely solely (as so many do) on algebra alone to transmit meaning:

> Visual figures in scientific text, and mathematical expressions also, are generally not redundant with verbal main text information. They do not simply "illustrate" the verbal text, they add important or necessary information, they complement the main text, and in many cases they complete it. (Lemke 1998. 105)

For Lemke it is not merely a matter of convenience or practicality:

> ". . . the different semiotic constructions that together and in relation to one another constitute "the concept" have *nothing* in common; there is no common denominator, and certainly no higher Platonic idea of which they are each pale shadows. It is in the nature of scientific concepts that they are semiotically multimodal . . . (Ibid. 111)

How are multimodal representations to be taught? To begin with, as with most of the material presented in this paper, merely alerting students to their existence is a significant step. Below are some suggestions from chemistry that apply to any scientific discipline:

> The development of representational competence can be fostered by explicitly engaging students in the creation of various representations and in reflection on their meaning. Students should be encouraged to represent chemical problems and solutions in a variety of ways and comment on how the representations are equivalent, how they are different, and why one form may be better at expressing a problem or solution than another for a particular purpose. Working in pairs or groups, students should be encouraged to use various representations as they talk to each other about chemistry—to describe, explain, question, and discuss their understanding as it is expressed in a variety of forms—for this is what chemists do. (Kozma and Russell, 1997, 965)

The authors suggestion a skill set that "might constitute the core of a substantive curriculum of representational competence in chemistry."

> The ability to identify and analyze features of a particular representation (such as a peak on a coordinate graph) and patterns of features (such as the shape of a line in a graph) and use them as evidence to support claims or to explain, draw

> inferences, and make predictions about relationships among chemical phenomena or concepts.
>
> The ability to transform one representation into another, to map features of one onto those of another, and to explain the relationship (such as mapping a peak on a graph with the end point of a reaction in a video and a maximum concentration in a molecular-level animation).
>
> The ability to generate or select an appropriate representation or set of representations to explain or warrant claims about relationships among chemical phenomena or concepts.
>
> The ability to explain why a particular representation or set of representations is more appropriate for a particular purpose than alternative representations.
>
> The ability to describe how different representations might say the same thing in different ways and how one representation might say something that cannot be said with another. (Kozma and Russell, 1997, 964)

These skills require little if any translation or modification for the subject of physics.

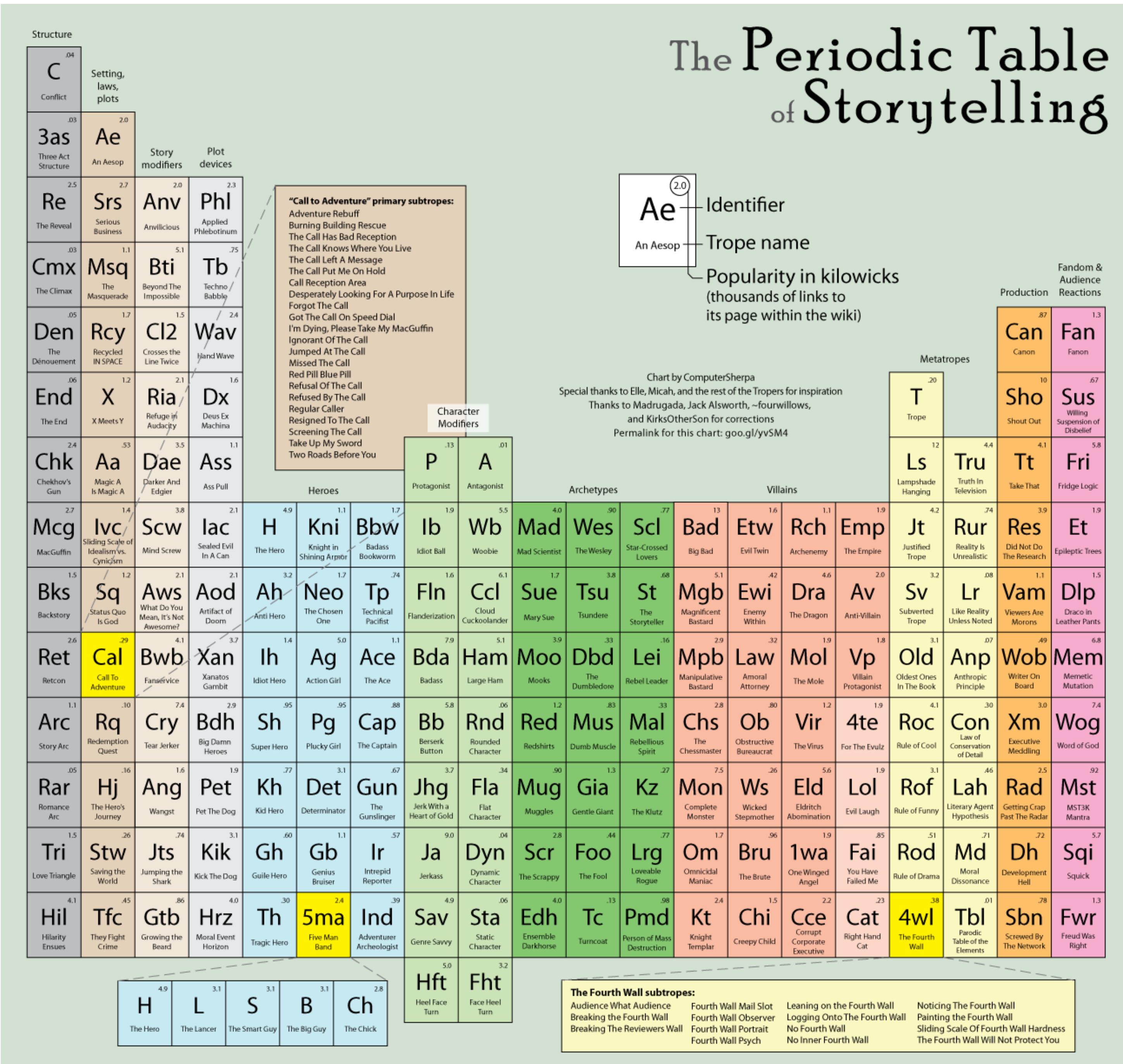
Concluding remarks

The language of science used to both construct and transmit meaning is a multimodal language, and the various modalities are not redundant, indeed their co-deployment serves to create and multiply meaning within a scientific text. Use of appropriate multimodalities within schemata clarifies and organizes students' knowledge and, more importantly, cues students to plot and thematic structure as well as the appropriate solution templates. For the marker student knowledge and intention are clearly revealed.

Far too many student solutions feature equations and diagrams splayed about the page, and though I can often piece together the plot in so doing I am invoking *my* knowledge of physics, *my* abilities as a problem solver, *my* experience in structuring the narrative: we should rather be marking the students abilities. We make copious use of graphs and diagrams in lectures and should encourage students to co-deploy several semiotics to arrive at a solution (or, in the case of conceptual questions, an understanding).

A Parting Shot

In the spirit of material presented in this section, I offer the following justification (without any accompanying verbal text) for the use of narrative elements in the mulimodal process of problem solution.[19]



[19] http://trojantopher.files.wordpress.com/2013/01/periodic-table-of-storytelling.png

5. Pedagogical Conclusion

The enemy of reflection is the breakneck pace—the thousand pictures.[20]

Jerome Bruner, The Culture of Education

Problem Solution in the Curriculum

To paraphrase an old English proverb,[21] a problem well presented is a problem well solved—it is not possible to decouple to two processes. In this essay I have used the term *problem solution* to refer to a process that begins with reading the problem and proceeds through the murder board investigation to the final explication that is the written solution. We use problems to teach concepts and evaluate student performance, but their value is far greater, for problem solving is *the* exemplary scientific activity: scientists spend the majority of their career not inventing new theories *ex nihilo* but rather in solving research problems.

> Problem solution is a central scientific activity, a vital skill that will remain with and enrich the intellectual life of our students long after they have forgotten much of the course content. We can and should use the physics curriculum as a vehicle for teaching this vital skill set.

The crucial skills of essay writing and argumentation have long been a part of the Humanities curriculum—it should be so with Science. And whether or not a particular student remains in the Sciences (or Humanities) is of no import, for the essential skills of writing and problem solution transfer to any domain.

[20] Bruner 1996, 129

[21] A problem shared is a problem halved.

Such curricular reform is not drastic and can be effected by a few additions and a change of emphasis, but for me it must begin with student cognizance:

There exists a body of knowledge about problem solution and students should be made aware that such research exists; awareness is in and of itself empowering.

The goal: student problem solutions, learning and knowledge organization that exhibit the expert habits detailed by *Physics Education Research*.

We should alert students to expert habits (and the pitfalls of novice habits) and explicitly include instruction in these habits as part of the curriculum: fundamental skills should not remain tacit knowledge.

Novice To Expert Progression

Can expert habits and skills be taught? There is a robust literature showing that indeed they can, though for some skills the road from novice to expert is a long one: many characteristics once believed to reflect innate talent are actually the result of intense practice extended over a minimum of 10 years (Ericsson et al. 1993). But a change in pedagogy can make skill sets available to those who exhibit little or no native talent.

Consider the skill of drawing. Some people have what seems to be an inborn talent for sketching: they can produce realistic or stylized sketches that transmit emotion, mood, etc, and have been able to do so for most of their lives. Contrast this to the majority who, like myself, seem to be capable of producing only childish scribbles. Can sketching be taught? Can someone like myself learn to *see* like an artist and to render my visions onto paper. The answer, according to the distinguished art educator Betty Edwards, is a resounding *yes*, but this answer came only after her initial frustration with the received pedagogy:

> Unlike many art educators who believe that ability to draw well is dependent on inborn talent, I expected that all of the students would learn to draw. I was astonished by how difficult they found drawing, no matter how hard I tried to teach them and they tried to learn. I would often ask myself, “Why is it that these students, who I know are learning other skills, have so much trouble learning to draw something that is right in front of their eyes? I would sometimes quiz them, asking a student who was having difficulty drawing a still-life setup, "Can you see in the still-life here on the table that the orange is in front of the vase?" "Yes," replied the student, "I see that." "Well," I said, "in your drawing, you have the orange and the vase occupying the same space." The student answered, "Yes, I know. I didn't know how to draw that." "Well," I would say carefully, "you look at the still-life and you draw it as you see it." "I was looking at it," the student replied. "I just didn't know how to draw that." "Well," I would say, voice rising, "you just look at it..." The response would come, "I am looking at it,” . . . (Edwards 1999, XI)

Of course when those proficient at drawing “look” they “see” something quite different from what a novice sees. After reflecting on her own habits as a talented artist—and a few chance encounters with cognitive theory and students copying pictures upside down—Edwards developed a methodology based on six basic skills (skills of seeing which are not reliant upon manual dexterity!) that has proven highly effective in teaching the art of drawing.

Edward’s experiences mirror those discussed above in the section *Problem Solving in the Lecture Hall: The Lie.* I repeat here Reif’s summary of how we teach problem solution:

> The most common approach involves exhibiting illustrative examples of problem solutions and then providing students with practice in solving similar problems. Occasionally some teachers suggest also a few helpful rules of thumb, while other teachers advocate predominantly student learning by independent discovery. (Reif 1981, 310)

If students persist in novice habits we essentially tell them to “just look at it” or “look harder” at the model solutions they have been shown. As with drawing so with problem solution:

> Such approaches are neither too effective nor efficient in furthering students’ learning of problem-solving skills, nor do such approaches lead to a cumulatively growing body of reliable knowledge about effective teaching methods. Indeed, teaching methods based predominantly on examples, practice and discovery are more primitive than those used in simpler domains (e.g. playing musical instruments or performing in sports) where many *instructors use explicit teaching methods based on a systematic analysis of underlying component skills* [my italics]. (Ibid.)

The Content of the Form: The Benefits of Narrative Problem Solution

1. narrative problem solution as a means to expertise

I am here advocating just such a "systematic analysis of component skills" by including the following in the curriculum: murder board analysis; the deployment of multimodal resources; solution templates and the narrative elements of theme, plot and motif. I must admit that I have been wary of explicitly teaching the narrative elements discussed in this essay—young students are too quick to cynicism. On the other hand when I do discuss solution templates toward the end of a one-semester *Introductory Mechanics* course I find the class to be unusually silent. Could it be rapt attention? I think so, and imagine that I do not delude myself, for the students want to do well in this course, mayhap feel overwhelmed by their studies and are grateful for such intellectual economies.

During the closing weeks of the semester I present the five basic themes of the course and—with a good problem in hand (*e.g.* the ballistic pendulum)—the recurring narrative elements of plot and motif and how they dictate the solution template (annotation, page layout and punctuation—yes, I use the mnemonic **app**). I find that I need not mention theme and plot as anything more than an analogy, though of course it is more than that. Rather I present, in the form of a course summary and review, a prescribed methodology for thinking one's way through a problem, what I call "thinking with a pencil:"

2. constructing meaning at the murder board

- playing with representations and translations among various modalities
- progressive reduction of diagrams
- recognition of the various narrative elements: theme, plot and motif
- try out a theme and see where it leads
- continue with an iterative process of extracting plot and theme from the story, playing with various multimodal representations, all of which invoke possible solution templates
    - allow the narrative elements (theme, plot and motif and solution template) to

dictate, or at any rate suggest, the method of solution

3. transmitting meaning in a written solution

Once a satisfactory solution has been reached at the murder board the first order of business is to state the theme, and not only—or even necessarily—using written text, as advocated by some (Leonard et al. 1996) but rather using a combination of modalities, most likely equation and text. Below is a list of the five themes from *Introductory Mechanics* (brief textual statements may accompany the equations).

Kinematics With Constant Acceleration	$\because$ the acceleration is constant we can use the kinematic equations
Dynamics Inertia Newton's Laws Torque	$\sum \vec{F} = m\vec{a}$ $\sum \tau = I\alpha$
Work/Conservation Of Mechanical Energy	There are no external forces doing work on the system and $\therefore$ $\boxed{\begin{array}{l} W_{ext} = \Delta E = 0J \\ \therefore E_f = E_i \end{array}}$
Conservation Of Linear Momentum	The net force on the system is zero, therefore linear momentum is conserved. $\boxed{\begin{array}{l} \vec{\mathbf{F}}_{net} = \frac{d\vec{p}}{dt} = 0N \\ \therefore \vec{p}_f = \vec{p}_i \end{array}}$ or $\vec{F}_{av} = \frac{\Delta\vec{p}}{\Delta t}$
Conservation Of Angular Momentum	The net torque on the system is zero, therefore angular momentum is conserved. $\boxed{\begin{array}{l} \tau_{net} = \frac{dL}{dt} = 0 \\ \therefore L_f = L_i \end{array}}$

One then employs the appropriate and necessary multimodal resources (most oft diagrams and equations) within the solution template (**app**)

To novice students it seems that each problem dictates its own unique solution template, but really there are a limited number of constituent elements (table, graph, diagram, equation etc.) used in a finite number of solution templates, reflecting the limited number of themes, plots and motifs. Rather than face a seemingly unlimited number of possible problems within a course one has reduced the list to a handful of exemplars—this is **conceptual economy**.

4. conceptual economy

Kuhn introduced the idea of conceptual economy in his groundbreaking study *The Copernican Revolution* (Kuhn 1957, 37 et. seq.) as a means of characterizing the significant scientific achievement that was the Greek Two-Sphere conceptual scheme of the universe.[22] The conceptual scheme is not just a theory but also the entire constellation of beliefs and commitments (psychological, religious/spiritual) that scientists bring to their discipline. There are various epistemological and ontological functions of conceptual schemes; here I consider the former as applied to pre-Hellenic astronomical knowledge.

- **the list**

  Since the time of ancient Mesopotamian civilization humans had complied an impressive list of complex, seemingly unrelated astronomical phenomena. For example, the sunrise and sunset points move northward on the horizon during the summer, then southward during winter (that is, from Winter Solstice to Summer Solstice and back again).

- **the conceptual scheme replaces the list (conceptual economy)**

  To one acquainted with its use, the model replaces the list of seemingly disparate observations, bringing to them order, coherence and meaning one with respect to the other—the various observations have now become *consequences* of the conceptual scheme. For example, the seasonal drift in sunrise positions is seen as a consequence of the sun's annual

[22] A *conceptual scheme* is the precursor to Kuhn's later *paradigm*.

sojourn around the ecliptic (a path which is tilted 23.5° with respect to the celestial equator). Thus the list need not be memorized: individual observations can be deduced from (are logical consequences of) the conceptual scheme.

- **explanation/understanding /confidence**

  The scheme explains, allows one to understand the regularity of the heavenly motions. Scientists armed with a well-tooled conceptual scheme can avoid the "paralysis of the abyss": the scheme tells them what to look for, where to look for it, and, for a given problem, even specifies the method of solution—would that all of our students could enter exams feeling so equipped. An adept can feel confident that the scheme, and one's own problem-solving ingenuity, will eventually yield a result.

the conceptual economies of narrative

Conceptual economies are inherent in scientific theories. Recognition of the various narrative elements discussed in this essay gives students access to the conceptual economies practiced by experts in scientific research.

- **the list**

  There are of the order of 100 exercises and problems at the end of each chapter in a standard undergraduate introductory physics textbook.

- **the conceptual scheme replaces the list (conceptual economy)**

  As we have seen, experts see unity where novices see diversity. For experts the list is reduced to a small number of what Kuhn terms *exemplars*. Kuhn defines exemplars to be "concrete puzzle-solutions which, employed as models or examples, can replace explicit rules as a basis for the solution of the remaining puzzles of normal science." (Kuhn 1970, 187) I think that all successful students have at some point made the critical intellectual transition that occurs when one realizes that there are but a handful of problems per course (exemplars), the remaining multitude mere variations on a theme.

  In the context of this investigation I take exemplars to consist of the entire narrative structure: theme, plot, motif and corresponding solution template[23]. There are a limited number of themes and plots associated with each course. Moreover, the themes, plots and motifs serve to structure the presentation of the solution: that is, the global semiotic of solution template is theme and plot dependent.

- **confidence**

  As with plot and theme, there are a limited number of solution templates that work quite well, with small modifications, for the vast majority of problems. Students who have a limited number of templates at hand—and are aware of the limited number of plots and themes—can enter an examination with what Mark Twain referred to as "the serene confidence of a Christian holding four aces." [24]

The narrative elements provide a framework for knowledge and mathematical skills to operate: it's like giving students a fully-structured essay outline, allowing them to properly focus their intellectual efforts. There is no need to "reinvent the wheel" with each problem, to face the abyss of an empty page. Composers (before modernity became all the rage) were quite content to pour their creative energies into the well-defined mold that was musical form (or style): rather than facing a blank score, Bach or Webern knew quite well what a fugue entailed and what harmonic and melodic language they would deploy within that style, etc. Similarly, there is a tremendous freedom of expression available within the rather rigid constraints of the 12-bar blues genre. And it is within the conventions of a genre that many students need to be, for too often novices seem to face the paralysis of the abyss (the void, having no idea where to begin) or of the plenum (the seemingly thousands of unique problems, Bruner's "breakneck pace—the thousand pictures").

explanation/understanding

As to the functions of explanation and understanding, expert problem solvers have a better

[23] There is currently a lively discussion interpreting Kuhn's historiography of science in light of recent developments in cognitive theory, such as schema theory. See, for example, Nickles 2003.
[24] attributed

grasp of physical theory (referred to in the literature as "domain knowledge") and know the utility and applicability of the various themes in problem solving. Novice students can only come to understand physical principles if they attempt to integrate them within a knowledge structure (a schema): narrative problem solution provides that structure. The narrative elements offer them a means (forces them, really) to access expert-level habits and to develop a conceptual understanding of the subject matter.

5. **various other benefits of teaching narrative problem solutions**

Pedagogy

- focuses pedagogy on the intellectual habits that we value and the fundamental role of conceptual knowledge in problem solution
- privileges communication of ideas (concepts) over results, and thus emphasizes the teaching of concepts
- provides for a systematic terminology to allow teachers and students to communicate about problem solutions, pedagogy and marking rubrics, and for student use to reflect on their own practices
- problem solution at the murder board—this is what scientists do, not just students, so we are in fact modeling scientific research

Diagnostics

- formal clarity can help reveal student misconceptions to the marker and, more importantly, to the student during the exam
    - many students are not really certain how to check their own solution. The solution template allows them to attend to details such as signs, units, errors in transcription, etc.
    - Experts are not just intelligent, they are careful, and the solution templates remind students of this important habit of successful scientists.
  - for teachers, such clarity allows us to see explicitly (rather than inferring, often incorrectly) student problem-solution procedures; how students understand conceptual knowledge and how it applies to problem solution

organization of, and reflection on, knowledge structures

- It is easier to digest new knowledge delivered in lectures when one has an effective tool for knowledge organization: "facts cling to a narrative."

> The task of creating such an effective organization is a substantially difficult undertaking which most students are ill prepared to carry our without outside assistance. (Reif 1981, 316)
>
> . . students tend to have one of two perspectives on learning physics: they either focus on getting the answer, or they work to have the physics make sense . . .

> sense-making and incorporating the understanding gained from doing so when solving a new problem leads to expertise. In contrast, focusing only on getting an answer leads to a restricted set of procedural skills. (Maloney 2011, 8).

> An individual's problem solving ability depends strongly on the organization, not just the extent, of her knowledge store. (Gerace and Beatty 2005, 5)

- increased retention: We remember and discuss stories not at the surface but deeper levels: it is easier to recall the general theme and plot structure of a narrative rather than the details of story. It is hoped that such an effect will obtain for narrative problem solutions.

> It requires great effort to recall the exact words used in a novel or the exact sequence of shots, angles, lighting, etc. used in a film. . . When we say we remember a film . . . when we speak of comprehending something, we mean that our knowledge of it may be stated in several equivalent ways; that is, our knowledge has achieved a certain independence from the initial stimuli. (Branigan 15)

organization of, and reflection on, study habits and exam preparation

- promotes more efficient and active study habits
- tests and exams are time limited, best to walk in with a few aces up your sleeve
- clarifies for students expectations in marking (more on marking rubrics below)

Pedagogical Varia

The multimodal language of science

We need explicitly teach how various modalities are co-deployed in problem solution to both construct and transmit meaning. There is too much tacit knowledge in science education—we miss important opportunities to transmit knowledge. A few evidentiary quotes:

> . . . any definition or principle should be accompanied by the learning of significant ancillary knowledge (including skills of symbol interpretation) so that any such definition or principle can be flexibly used."(Reif 1981, 314)

> . . . many advocate teaching learners metacognitive strategies designed to activate one's schema before reading, such as reading heading and the title, looking a visuals in the text, and making predictions based on the title and pictures" (Widmayer 2005, 2)

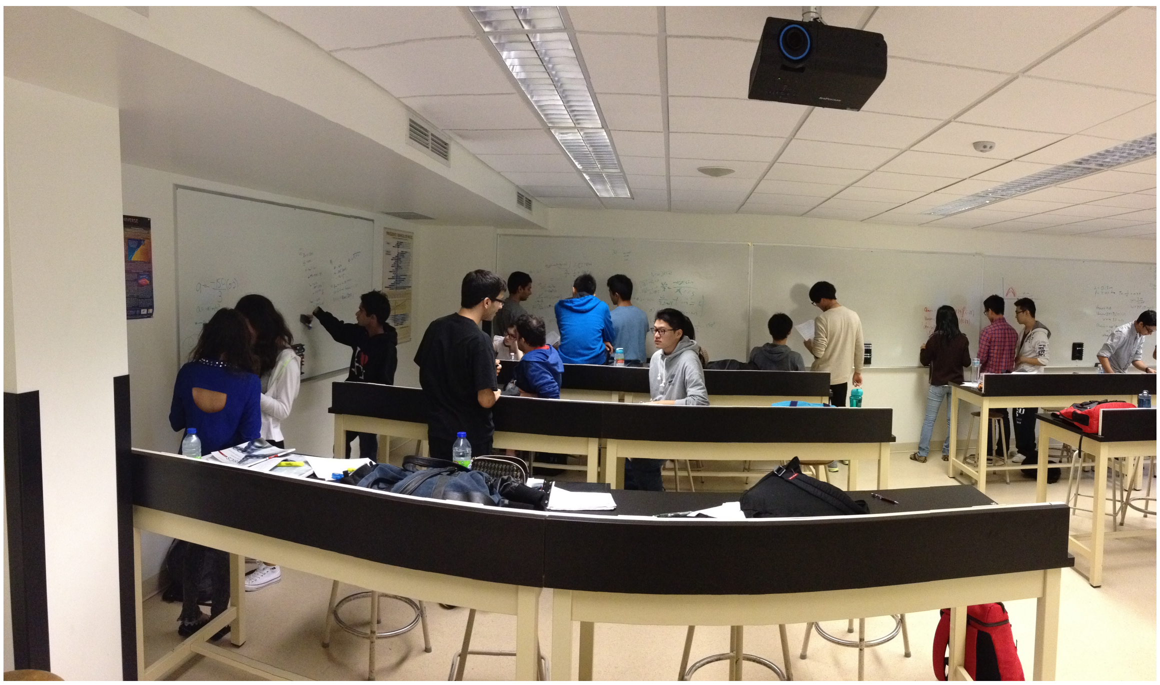
> If students are asked on an exam to represent a situation in multiple ways without solving for a particular quantity, they will understand that the ability to re-express concepts has value. Another way to achieve the same goal is to provide students with problems that are difficult to solve without representing the

situation in multiple ways (Etkina & Van Heuvelen 2008, 25)

conceptual questions

Multiple choice concept questions should be approached in a manner no different from problems. Their solution requires students to employ the full arsenal of scientific problem solving skills: pure mentation is insufficient.

the murder board



- Physical layout of classrooms: one of our physics labs has white boards on three of the four walls. Students solve problems at the board in small groups of 2-3. I circulate with coloured markers and help annotate, punctuate, cajole, hint, correct etc. The students have found this to be a highly productive use of time, and in the past several have bought whiteboards for their home.
- During lectures we should model realistic problem solving at the murder board, making mistakes, mucking around, modeling getting stuck and unstuck, talking about this

(anecdote about good/poor student, determined what error he made, will never make the error again—good students make fewer errors but more mistakes) write, scribble sideways, erase, start again etc.

- When solving a problem on board I often (and not always intentionally) make mistakes to model getting unstuck. Often on board or iPad (they see projected image) I write sideways, a way of decoupling my thinking on paper process from the writing of the solution.

marking rubrics

Clear statements of theme and a well-wrought solution template ease the marking process, also making it more effective. Students can be given the marks they deserve, based upon what they know and how they communicate this, not merely the result of a calculation (at any rate a correct solution is can often be based upon faulty theory)

> Ability to provide lucid, logical explanations is highly valued in our profession . . . Basing student grades solely on problem-solving performance is tantamount to evaluating only a subset of skills that we value in our peers. (Leonard et al. 1996, 1502)

- We should only "make meaning" with what is presented to us on the page and *never* invoke our knowledge to fill in the gaps
- teach it, insist that they use it, mark it—they will take it seriously

> . . . faculty often have difficulty following through on assigning low scores to students' solutions that do not contain explicit communication of component processes such as a diagram if the "answer" is correct. In addition, instructors have a tendency to assume, if the answer is present, that students have used appropriate reasoning or processes even if the students do not communicate the processes and reasoning they employed. Maloney 2011, 22)

- In an unguarded moment during a lecture I actually said, "It's not what you know, it's what you show." Rather embarrassing really, but it was all very spontaneous—I noticed several students writing it down.
- Assign marks for correct identification of theme (most teachers probably do this) but also

for correct identification of other narrative features (plot, motif) and **app.**

- The distinction between reader and writer is important: teachers are readers of student texts, and we need to know that students understand and are not blindly plunging into algebraic problem solving (as in the first student example given in the Introduction).
- We should not reward students for relying upon novice habits. For example, novices often begin a solution to interference problems with an algebraic statement that conflates geometry ($\Delta r \approx d\sin\theta$) and physics ($\Delta r = m\lambda$) in shortcut to numerical answer:

  $$d\sin\theta = m\lambda \,.$$

  Rather we should insist on a full problem solution, something like the following:

  For a maximum

  $$\Delta r = m\lambda$$

  where $\Delta r \approx d\sin\theta$

  and thus

  $$d\sin\theta = m\lambda \,.$$

  Students accustomed to this fuller solution statement will be more likely to solve the following:

  In a double-slit apparatus with light of wavelength 600 nm, how much farther is it to the third maximum above the central axis as measured from the bottom rather than the top slit?

  No geometric approximation is required, the problem is one purely of interference (theme) generated by path difference (plot):

  $$\Delta r = m\lambda$$

  where m = 3.

- deduct marks even if the solution is numerically correct:
  - *e.g.* 1 *A mass attached to string ascends with a constant velocity.* The statement of Newton's Second Law must convey information (the fact that $a = 0$)

$$\sum F_x = ma_x$$
$$T - mg = 0$$
$$not$$
$$T = mg$$

- *e.g. 2 C*onsider the equation for image formation by refracting surfaces:

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

A full problem solution, even to a simple *plug-and-chug* exercise, must specify the sign of *R*: when substituting numbers into the right-hand side of the equation experts will communicate their understanding:

=(1-1.5)/(-3)

Novices will simplify on the fly—they are only concerned with the numerical answer:

=0.5/3

One Last Parting Shot: aesthetics and explanation

It has been said that poetry reminds us of the value of lingering. I well recall a lecture in Group Theory when, having completed a proof of one of the Sylow Theorems, the teacher stepped back from the board and sighed. In fact we all did, for the proof was a thing of beauty. The opportunities for such lingering are rare and most likely cannot be contrived—we should seize them when they occur. To one so inclined the precise and powerful language of science affords such opportunities. Consider the derivation of the law of refraction using *Fermat's Principle*:

> You could validly argue that the minimum formulation is neat, but really no better than the other formulation. However, move from this lecture room to your bathtub and observe your big toe in the water. Your limbs no longer appear straight because the velocity of light in water differs from that in air. The least-time principle tells you how to formulate behaviour under such conditions and the memorizing of Snell's Law about angles does not. Who can doubt which is

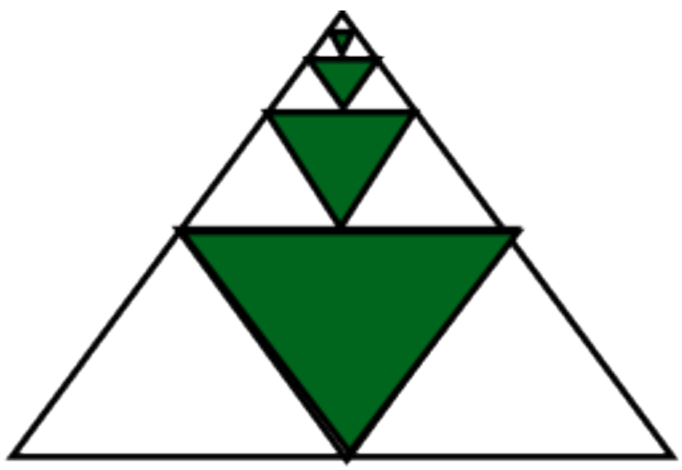
the better scientific explanation[25]?

Good solutions are aesthetically pleasing, and we do value beauty and simplicity in mathematics and physical theories (Occam's Razor):

> The research worker, in his effort to express the fundamental laws of Nature in mathematical form, should strive mainly for mathematical beauty. It often happens that the requirements of simplicity and beauty are the same, but where they clash the latter must take precedence. (Dirac 1939, 122)

explanation: appropriate choice of theme

The outer equilateral triangle in the diagram below has an area of 1 unit.



There are two methods to determine the area of the shaded equilateral triangles (an infinite array of inverted triangles inscribed such that their vertices bisect the sides of the larger triangle in which they are inscribed). I gave this example to students at a high school for the gifted: the grade 12 class dutifully and correctly summed the infinite series, as they had been taught to do. The grade 7 class had been spared instruction in series and so took to lateral thought—literally. Several students saw at once that along each row 1 in 3 triangles is shaded, and thus the total area of the inverted triangles is 1/3. Nice!

[25] Paul A. Samuelson, *Maximum Principles In Analytical Economics* Massachusetts Institute of Technology, Cambridge, Massachusetts Nobel Memorial Lecture, December 11, 1970.

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